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THESIS

Composite Reliability:
Determination of Fiber Strength
via Bundle Testing

by

Joseph Albert Schmidt

September 1988

Thesis Advisor: E. W. Wu

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The values obtained from the bundle experiment as compared to known single fiber test bench mark parameters were practically indistinguishable. The confidence of the results stem from a thorough analysis of the bundle mechanisms and minimized contaminates which can disturb the strength distribution.

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Composite Reliability:
Determination of Fiber Strength
via Bundle Testing

by

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Lieutenant, United States Navy
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Submitted in partial fulfillment of the
requirements for the degree of

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from the

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ABSTRACT

Composite reliability is strongly dependent on the fiber strength distribution. Current methods of gathering statistics through single fiber failure methods are inefficient and costly. This thesis develops a testing method from which the fiber statistics in the form of Weibull parameters can be accurately extracted from bundle failure tests.

The values obtained from the bundle experiment as compared to known single fiber test bench mark parameters were practically indistinguishable. The confidence of the results stems from a thorough analysis of the bundle mechanisms and minimized contaminates which can disturb the strength distribution.

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I. INTRODUCTION

A composite, by definition, is any two or more materials combined on a macroscopic scale to form a useful material [Ref 1]. Today, in one form or another, composites are being used on every level of our society. This thesis will focus on a class of composites known as "fibrous composites," today these are typically used in high tech applications, often high performance military aircraft. Fiber composites consist of very thin fibers with high aspect ratios and high strengths imbedded in a matrix, commonly a polymeric matrix such as epoxy. Glass/epoxy is an example of a frequently utilized composite mixture that has been in use for many years.

One of the many advantages this class of material provides is high strength-to-weight, though due to a lack of thorough understanding in failure mechanics, composites have been limited to secondary load carrying structural designs such as skins and control surfaces. In recent years (last 20), composites are expanding into a much wider role.

A. REQUIREMENTS IN ADVANCED STRUCTURES

Since the introduction of glass/epoxy, advances in production techniques and developments in the textile industry have resulted in fiber materials with much improved strengths and stiffness. Two of the early break-throughs were graphite and boron fibers which have typical strengths and stiffness much higher than convention ductile isotropic metals. These fibers allow for new

design limits but also give rise to the requirement of new design technique utilizing a material of an unconventional material redundancy with anisotropy.

Employing the fiber/matrix as unidirectional lamina to form multidirectional laminate gave the designer the tools to make structural designs subjected to multiple loading conditions a manageable task. This idea shifted the attention of the designer from the microscopic level to the macroscopic level.

Utilizing the high strength and stiffness, fibers application for structural enhancement were incorporated in many of the military designs in the early 70's such as the F-111 aft fuselage structure. Testing of these designs were accomplished by component. This method is time consuming and expensive but must be done. To gain any type of reliability assurance many tests are required. It soon becomes apparent that testing costs could out pace production costs.

B. FUTURE DESIGN REQUIREMENTS

As weaponry becomes more and more sophisticated the demands on composite materials for a wide variety of uses are ever increasing. New designs often are only limited by the structural limitations of the materials in use. Such restrictions might include the thickness of a wing or stabilizer, the sweep of the wing, or the "g" loading. Lately new design specifications have included the use of composites for the radiation energy absorbing properties. Though the Navy's interest in composites spans a multitude of uses, one special interest is in the area of structural enhancement, such as rocket motor cases, pressure vessels for submarine flasks and jet aircraft pilot ejection seat.

In the last few decades composites have advanced dramatically, and are beginning to provide improved alternatives to more conventional methods of design normally utilizing isotropic alloys. Fiber reinforced plastics have attracted a large amount of attention. Improved techniques in the textile industry have lead to more uniform fibers from bases of precursors for high performance graphite's with strengths and stiffness exceeding high yield steels.

As utilization goes up the increased use of new fibers in composites and the cost of system failure testing demand better methods for reliability evaluation.

II. BACKGROUND

A. COMPOSITE LOAD SHARING

A brittle material generally has a higher strength to density ratio but a lack of ductility limits the structural uses. Ductility resists crack propagation from a region of damage. Metals have a high ductility and do provide practical strength and stiffness properties for many structural applications. Fibers do not use ductility to resist failure [Ref. 2:pp. 1-19], because ductility is not required the fiber can be synthesized from brittle materials.

The maximum theoretical elastic strain for carbon hydrogen bonds is approximately $\varepsilon_{max} = 0.1$, which is one order of magnitude greater than present day strong elastic reinforcing fibers.

Presently fibers can not obtain strengths any where near the theoretical ideal strength of the material but are an order of magnitude greater than conventional bulk materials. Fiber strength can also be attributed to the small fiber diameter, which limits the flaw sizes.

Composites are geometrically advantageous to homogeneous material such as an all metal or all ceramic material, for its ability to utilize the high strengths and stiffness of brittle materials while maintaining strength redundancy through the matrix.

The strength interaction of the fiber and matrix is the key to reliability effectiveness of a composite. Consider several fibers in parallel without matrix. The load is equally shared by each fiber. If one fiber should fail the failed fiber is completely ineffective and no longer supports any load. The load of the

remaining fibers increases equally. With a matrix present (Figure 1) a crucial phenomenon occurs. At the point of fracture the more ductile matrix absorbs the load through shear stresses along adjacent fibers and back to the broken fiber itself. Near the fracture site high shear stress develope in the matrix along the broken fiber but quickly dissipate as the normal stress increases. On the two surrounding fibers a noticeable stress increase is observed adjacent to the fracture. The short distance from the broken end of the fiber to the point that fiber is carrying a full stress is called the ineffective length [Ref. 3]

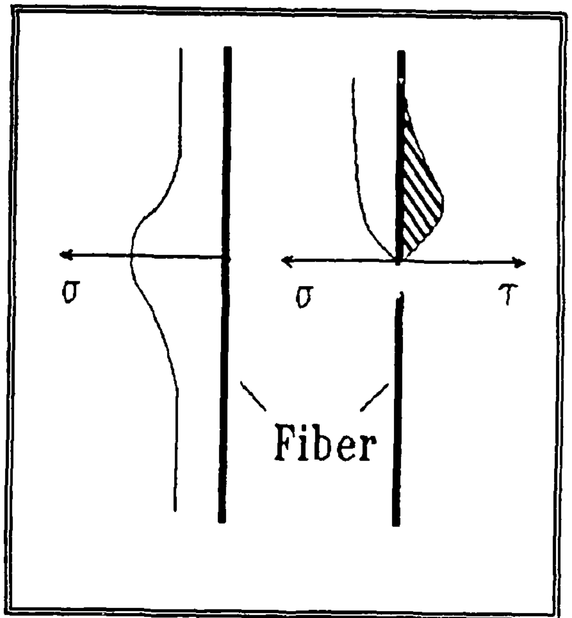

Figure 1. Local load sharing in a Fiber/Matrix composite.

The ability of the composite to adjust to multiple fiber failures is it's strength redundancy, and is dependent on the characteristics of both the matrix and fiber. There are several fracture mechanisms all of which are a function of the bond strength (shear strength of interface), the matrix shear strength and the distribution of the fiber strength. The most critical is the fiber strength distribution. The distribution of the fiber determines the composite reliability.

B. STATISTICS AND RELIABILITY

The complexities of composites make the task of analysis virtually impossible. New fibers are being developed every year and attempting to generate thorough material testing studies as has been done with conventional materials is not realistic. But composite reliability can be estimated through probability studies. Probability of the composite must be based or inferred from accurate statistics of the fiber distribution.

To determine the reliability of the composite, the strength distribution of the fiber must be known. The distribution can be quantified by strength statistics (a Empirical Probability Distribution Function (EPDF), or Empirical Cumulative Distribution Function (ECDF)). For computational efficiency the empirical data can then be modeled with an analytical function and thereby characterize the distribution by parameters of the function. To define the parameters the distribution function must be identified by model type (i.e., Normal, Weibull, etc.). Using the model Probability plot; the closeness of fit of the data can be judge and therefore the appropriateness of the model. Selecting the model allows for the calculation of the model parameters.

The relationship between the fiber statistics and the composite probability is represented in a linearized Probability plot (Figure 2).

Reliability is mostly concerned with the lower tail of the distribution, this also being the most difficult to define. Because of the redundancy provided by the matrix induced load sharing the composite is considerably more reliable than it's constituent fibers in the lower tail. As the fiber statistics fluctuate in the extreme lower tail the composite reliability fluctuates with it. This can be

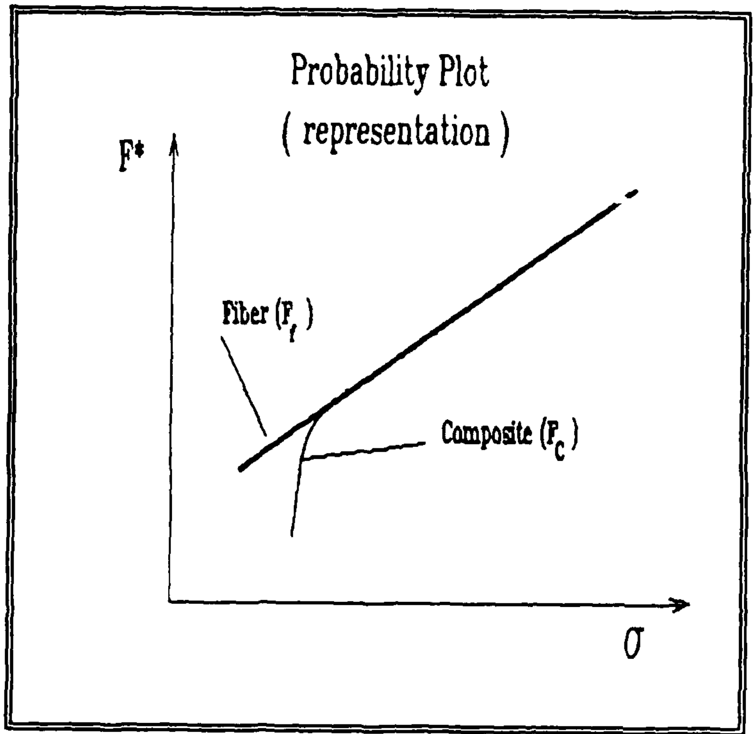

Figure 2. Comparison of Fiber and Composite in a Probability Plot.

shown in a representation of a probability plot in Figure 3. There is a critical point where the fiber plot stabilizes, the linear region in this plot determines the accurate statistics of the lower bound reliability (worse case) of the composite.

Fibers have a characteristic which helps to identify the analytical distribution model that will closely define the empirical statistic. Traditionally, the weakest link model [Ref. 4] depicted the fiber as being made of many small segments linked together much like a chain. The segments each have an intrinsic strength, none being exactly equal. When the fiber is stressed the weakest will fail and the total fiber fails. The Weibull distribution model

defines this weakest link behavior and is commonly used in modeling fiber strength.

1. Weibull Density Distribution Function

The Two parameter Weibull cumulative distribution function (CDF) is defined by the following equation.

$$F(x) = 1 - \exp\{-(x/\beta)^{\alpha}\}, \quad x, \beta, \alpha > 0 \tag{2.1}$$

where: x = random variable (stress, load)
β = scale
α = shape

The parameter alpha defines the shape (spread and skew) of the distribution. Alpha less than 3.5 the plot is skewed positively, greater than 3.5 it is skewed negatively and neutral at 3.5. The alpha parameter determines the strength scatter of the fiber, an increasing alpha defines less scatter and higher strength uniformity. The beta parameter defines the central tendency of the distribution and is relatable to the mean. Alpha for a fiber type is constant for the weak link physics, whereas beta changes with the fiber length. An increase in length causes a decrease in beta (a weaker fiber).

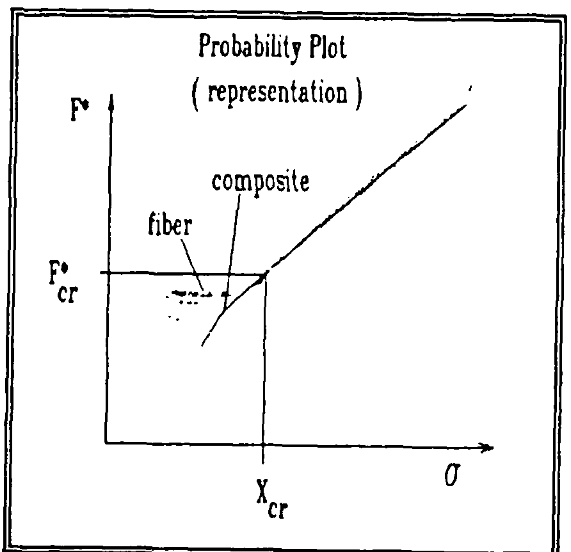

Figure 3. Extreme lower tail.

The composite reliability can be inferred from the distribution of a fiber strength. To gain high reliability with high confidence levels a proportionate amount of statistics must be obtained (i.e. for a reliability level of 10^{-3} requires 10^{4} statistics).

Presently fiber statistics are gathered by single filament testing (Figure 4). First a single fiber specimen is prepared, since a fiber has such a small diameter, (AS-4 diameter $\simeq$ 10 micron) it is very fragile. The fiber is placed in a cardboard frame. Once the specimen is loaded in the test apparatus the cardboard frame is severed by a heated wire followed by load application and interpretation of the load deformation data. This is a time consuming effort which produces only one failure strength statistic. To gain the number of statistics necessary would take thousands of tests, as an example man-safe reliability is 10^{-6} therefore to achieve the necessary statistics would require 10^{7} tests.

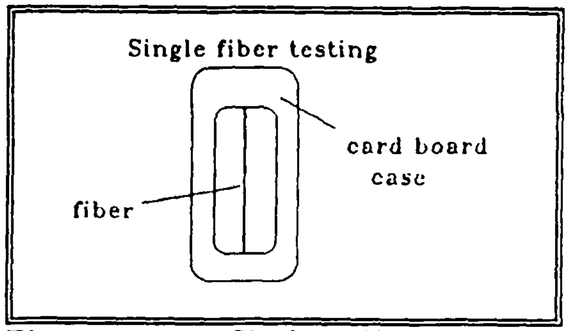

Figure 4. Single filament test specimen.

A new approach being explored is to test many fibers in parallel (a bundle) and recover the fiber characteristics from the failure test. This would provide a large statistical base required and could be accumulated with a much higher efficiency. With a validated method the analysis of fiber reliability could keep pace with the development of new fibers with reduced effort.

2. Size Effect.

The extreme lower tail is difficult to determine accurately for the fiber. For a Weibull failure physics and the resulting strength distribution the lower tail can be estimated by assuming weakest link [Ref. 4]. The parameter beta is a function of the fiber length and is defined by equation 2.2

$$\beta_2 = \beta_1 (l_1/l_2)^{(1/\alpha)} \tag{2.2}$$

where β_1 = beta for l_1
β_2 = beta for l_2
$\alpha = \alpha_1 = \alpha_2$

A detailed development can be seen in Appendix A. Using this relationship parameters for beta can be retrieved analytically where practical testing is impossible and the extreme lower tail is estimated.

This thesis will attempt to measure the lower tail via bundle testing, as opposed to the aforementioned estimation, (fiber statistics via single filament testing). This investigation explores the methodology needed in testing a bundle of fibers to failure. By recording the load and displacement it is expected that the underlying distribution of the fibers in that bundle may be extracted. If, for example, a bundle has 3k of fibers, the distribution is based on 3k of statistics. As the statistical base approaches the complement of the reliability, the accuracy of the estimation increases.

III. TECHNICAL ISSUES AND RESOLUTIONS

A bundle of fibers tested to failure presents several experimental obstacles which effects the analysis. There are three experimental consequences to testing a bundle in parallel that are not experienced in testing a single fiber in series. These phenomenon which may disturb the failure distribution, are 1) a protective/adhesive enhancement coating called sizing which is placed on the filament spools during production, 2) friction which occurs during testing after a portion of the bundle has failed and 3) slack which results from specimen fabrication. Each of these is discussed in the following paragraphs.

A. SIZING

Fibers are packaged on spools of bundles, in order to prevent the fibers from entangling a small amount of a chemical liquid called sizing is coated on the bundle. The effect of this sizing on bundle testing is unknown and must be address at sometime. The testing in this experiment was done leaving sizing on the bundle. A method to remove the sizing chemical without damage to the fibers needs to be studied but is not addressed here.

B. FRICTION

During the bundle test prior to fiber failure there is negligible friction, this is due to the equal strain between fibers. As the fibers fail the strain of the failed filaments return to zero leaving the loose ends entangled in the remaining fibers, this places added strain on those fibers. The resulting effect shows up as reduced failure strengths and clustering of the these failures. In

the relatively short gauge lengths friction is expected to be small and very pronounced in the long gauge lengths. The effects of lubricating the bundle with oil during testing will be observed to see if the effect is reduced.

C. SLACK

During specimen preparation and placement in the test apparatus the alignment of the fibers becomes misaligned with unequal length. The effective result is an initial nonlinear load/displacement which also effect the failure distribution. Another concern is the deformation in the actual load train of the test apparatus. This is referred to as system compliance. Compliance was found to be a function of the load and can be removed from the resulting data.

IV. EXPERIMENTATION

The experimental setup consisted of a Material Tester (INSTRON 4206) and an integrated data acquisition system consisting of an Instron Control Console, IBM PC/AT and software, a detail description and a list of procedures is in Appendix B. The object of the experiment was to test a current composite fiber type (used in many Navy aircraft) with a range of several gauge lengths and attempt to extract the underlying distribution of the fiber. The sample preparation is described in Appendix B. Each bundle sample was tested, the data stored, and quick analysis conducted to ensure no improper testing method were corrupting the data.

A. BUNDLE TESTING

The fiber material tested was made from a Hercules Magnamite high strength graphite, type AS-4 spool 145 in 3k bundles (i.e. there are 3,000 filaments in a bundle). Several different gauge lengths were tested in order to analyze the effects of friction, slack, compliance on the fiber size. Size affect (Appendix A) should be able to correlate the location parameter beta between fiber lengths if weakest link physics is applicable. In all, nineteen bundle samples where tested. The test procedure was conducted as specified in Appendix B. The complete test log is listed in Table I.

The bundle samples were handled with extreme care so as not to damage the any of the fibers. Moving the bundles from assembly bench to the storage bench and then to the testing machine presented the greatest hazard. The difficulty came when picking up the sample. This was done manually and

Table I. TEST LOG.

Sample #	G.L. (cm)	Oil/ Dry	Comments
090901	5.0	dry	Good
090902	0.5	dry	No failure-slip
090903	0.5	dry	No failure-slip
090904	0.5	dry	No failure-slip
090905	0.5	dry	No failure-slip
090906	0.5	dry	No failure-slip
090907	5.0	dry	Slip
090908	5.0	dry	Good
090909	5.0	oil	Good
090910	2.5	dry	Possible damage
100901	2.5	dry	Good
100902	2.5	oil	Slip
100903	2.5	oil	Slip
100904	50.0	dry	Good - (friction)
100905	50.0	oil	Good - (friction reduced)
100906	50.0	oil	Possible damage
100907	25.0	oil	Good
100908	25.0	dry	Good

bending the bundles slightly was unavoidable. The long bundles (500 and 250 mm) were less vulnerable to damage because of the relatively large aspect ratio but these were difficult to steady during transportation and tended to bounce around.

Placing the sample in the grips also presented operational difficulties when tightening the upper grip with a socket wrench. An attempt was made to manually counter torque the grip, but this tended to be jerky at best. Also the grip pressure was difficult to control between samples. The alignment of the bundle in the grips was another concern. This was done by sight. If not done properly the bundle failure mechanism might be influenced. Manually reducing the experimental slack (not to be confused with bundle slack) was very touchy. When toggling the grips at the control console the load read out had to be

acutely monitored for the bundle loading must be kept to a minimum prior to testing.

During testing a real time graphical display of the load vs displacement was used for monitoring the results, with this experimental irregularities in the several of the samples was detected. The bundles in the very short gauge lengths showed irregular load curves. Keying into this after several samples helped to identify a problem with the sample preparation.

Friction was expected to affect the long gauge length (250 and 500 mm) and not affect the shorter gauge lengths (25 and 50 mm). To test the affect each gauge length tested a pair was treated with oil and a pair was left untreated (dry). Tests were run on samples of 500 mm, 250 mm, 50 mm, 25 mm and 5 mm.

B. DATA REDUCTION

After tests were complete the data files were decoded into ASCII and stored for processing. The data output from the Instron software is a three column file. The first column is the record number, the second is the displacement (inches) and the third is the load (lbm). This is then processed through a program to convert the data to mm and kg and remove compliance. There are two data files output, one file with compliance removed and the other converted raw data (compliance not removed) [Ref. 5]. The output from the conversion program is then processed through an analysis program listed in Appendix C. This outputs the ECDF, the bundle Modulus $\underline{E}$ and the Slack region ECDF. The ECDF is then processed through a Weibull Maximum Likelihood Estimator program to define the shape parameter alpha and the

location parameter beta. ECDF is also processed through a linearizing program which outputs the Weibull probability plot data file.

V. RESULTS

A. TEST OBSERVATIONS

During testing several of the specimens were invalid due to experimental difficulties. Some of the samples had maximum strains noticeably beyond expected limits. One bundle was observed to have a much lower modulus than what was expected from equation C.5. Dissection of the samples with suspect loading curves revealed a lack of adhesive wetting resulting in a large percentage of the fibers slipping from the tab. The problem appeared most frequently in the small gauge length (GL) Samples. It is suspected that the adhesive which couples the bundle and tab was not wetting the internal fibers of the bundle, with a resulting loss of ability to carry the higher loads of the small GL. This phenomenon manifested itself two ways. If there was partial slipping after initiation of the failure region, a strain higher than the actual maximum <u>strain</u> was observed. If total slipping occurred in a large group of fibers the <u>modulus</u> decreased. After closer inspection of all the samples it was seen that many of the specimens had a small number fibers slipping in the tabs but the percentage was small enough not to effect the results.

To avoid this problem the remaining tests were conducted in samples at or beyond 25 mm. The problem was less acute in the long GL because of the lower loads these long GL's experience as a consequence of size effect.

Two types of test were carried out, with oil and dry. This data was then reduced by categories of compliance removed and not removed. The compliance was estimated by testing samples of zero GL [Ref. 5]. The resulting curves were fitted and reduced to an analytical equation, displacement as a function of

load. This equation can then be used to subtract the compliance from the empirical data. The zero GL samples used the same coupling adhesive and possibly suffered from partial slipping. This causes the compliance curve coefficients to be over estimated and as a result displacements removed for compliance were too large. When the compliance was removed from the experimental data displacements in the lower range showed negative values indicating over estimations.

Since the determination of compliance is slightly in error it is not used in the results discussion, though data reduction calculations were completed to indicate possible trends and affects. Table II shows the data reduction with removal, Table III is without compliance removal and Table IV, for comparison, is the result from single fiber tests [Ref. 6].

Table II. EXPERIMENTAL DATA RESULTS, COMPLIANCE REMOVED.

Compliance Removed

Test # dry/oil	Gage length (mm)	Modulus (kg/mm)	Modulus (gm/ε)	alpha	Beta (mm/mm)
100901 dry	25	109.76	915	3.0	0.0157
090901 dry	50	64.09	1068	3.214	0.0135
100908 dry	250	11.0	917	3.18	0.00972
100904 dry	500	5.433	905	3.035	0.0076
090909 oil	50	54.7	912	4.465	0.0145
100907 oil	250	11.18	932	3.964	0.0101
100905 oil	500	5.755	959	3.66	0.0085

B. DATA COMPARISON

Generally the results indicate only small deviations in the alpha parameter (Table III) for all GL. Between a oiled sample of the same GL only the 500 mm GL had any noticeable change in the shape parameter, which had been expected. The location parameter was effected most by the oil treatment, beta's increased by 10% in both the 50 and 500 mm GL. In 250 mm GL the alpha and the beta parameters seem to be relatively stable, meaning the oil treatment had little effect.

It was expected that friction would affect the longer GL's and have little or no affect on the shorter samples. The 500 mm test did show a considerable difference when oil was applied (Figure 5).

The friction was expected to become influential only as the percentage of failed fibers increased, but the dry bundle shows a decreased load early, which can be seen in the ECDF plot (Figure 6). In fact the oil and dry sample data merge as the percentage of failures increases. The reduced friction made a difference in the resulting Weibull parameters, alpha increased from 3.085 to 3.8. From single fiber testing alpha (Table IV) for AS-4 is approximately 4.11. Even with oil the bundle is affected, as seen by the sudden drops in the bundle load (Figure 5) or the sharp increase in the percentage of failures (Figure 6) at discrete points.

Without friction the failure distributions are expected to be smooth. Decreasing the GL it is anticipated that the effects of friction diminishes. Tests at 250 mm the loading curve is seen to smooth out (Figure 7), providing a slightly more continuous plot. Friction had less effect on this GL though a slight decrease is noticed when the bundle is dry.

Table III. DATA REDUCTION, COMPLIANCE NOT REMOVED.

Compliance Not Removed

Test # dry/oil	Gage length (mm)	Modulus (kg/mm)	Modulus (gm/ε)	Alpha	Beta (mm/mm)	Beta sized to 50 mm
090901 dry	50	45.6	761	4.75	0.01615	0.01615
100908 dry	250	9.85	820	4.07	0.0104	0.0154
100904 dry	500	5.2	867	3.085	0.00781	0.0137
090909 oil	50	43.5	725	4.89	0.0179	0.0179
100907 oil	250	10.6	833	3.95	0.01041	0.0154
100905 oil	500	5.5	917	3.8	0.00868	0.0152

SUMMARY	α	Std	ß	Std.	E	Std
OIL	4.21	0.59	0.0162	0.0015	825	96
DRY	3.96	0.84	0.0151	0.00125	816	53
Merged	4.09	0.66	0.0157	0.00137	821	75

Table IV. RESULTS FROM SINGLE FIBER TEST AT 50 mm GL.

Fiber Load Weibull Parameters

Spool #	Alpha	Beta
008	4.28	0.0175
019	3.94	0.0172
average	4.11	0.0173

Modulus = 900 gm/ε

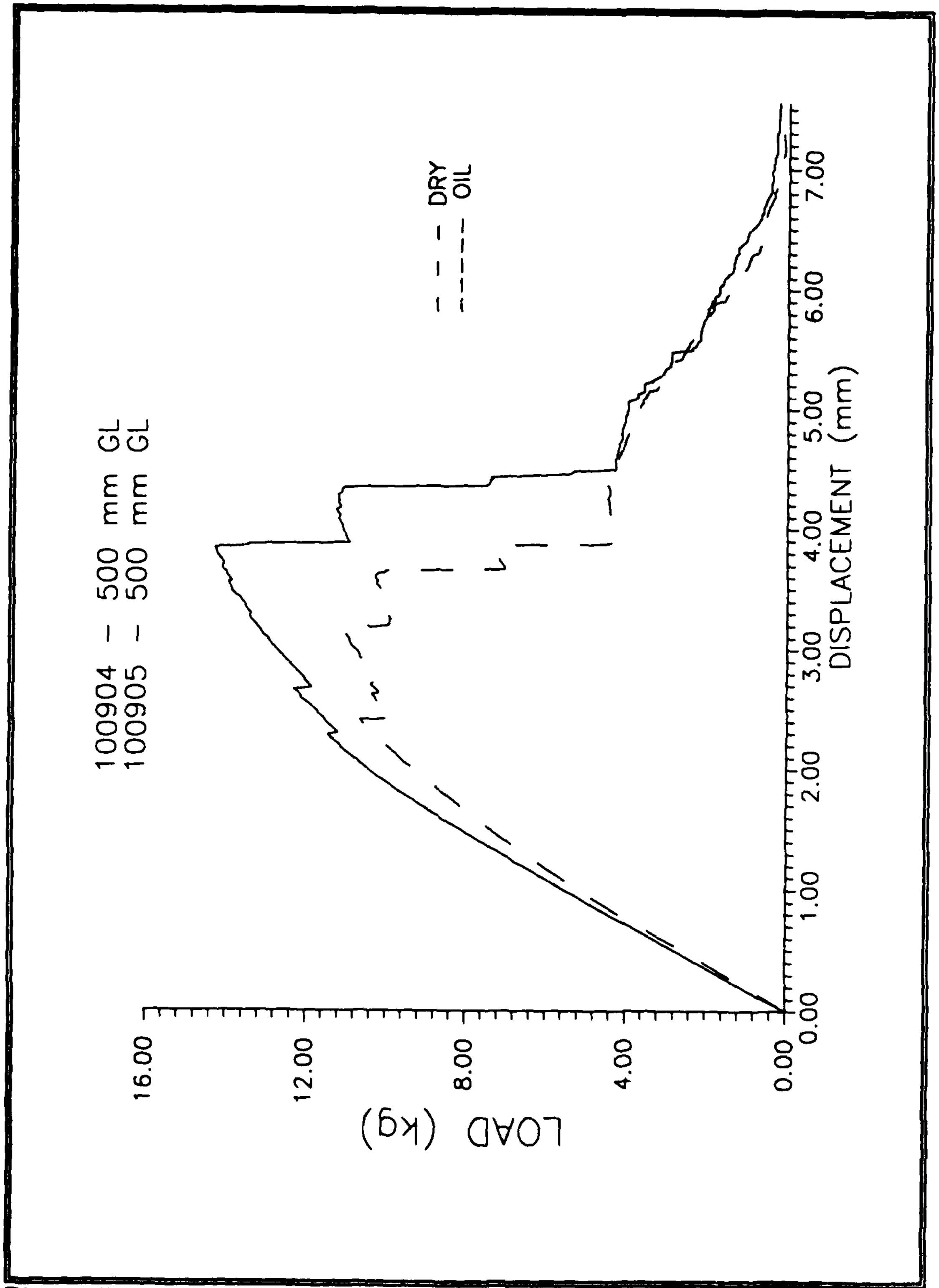

Figure 5. 500 mm gage length load curve comparison.

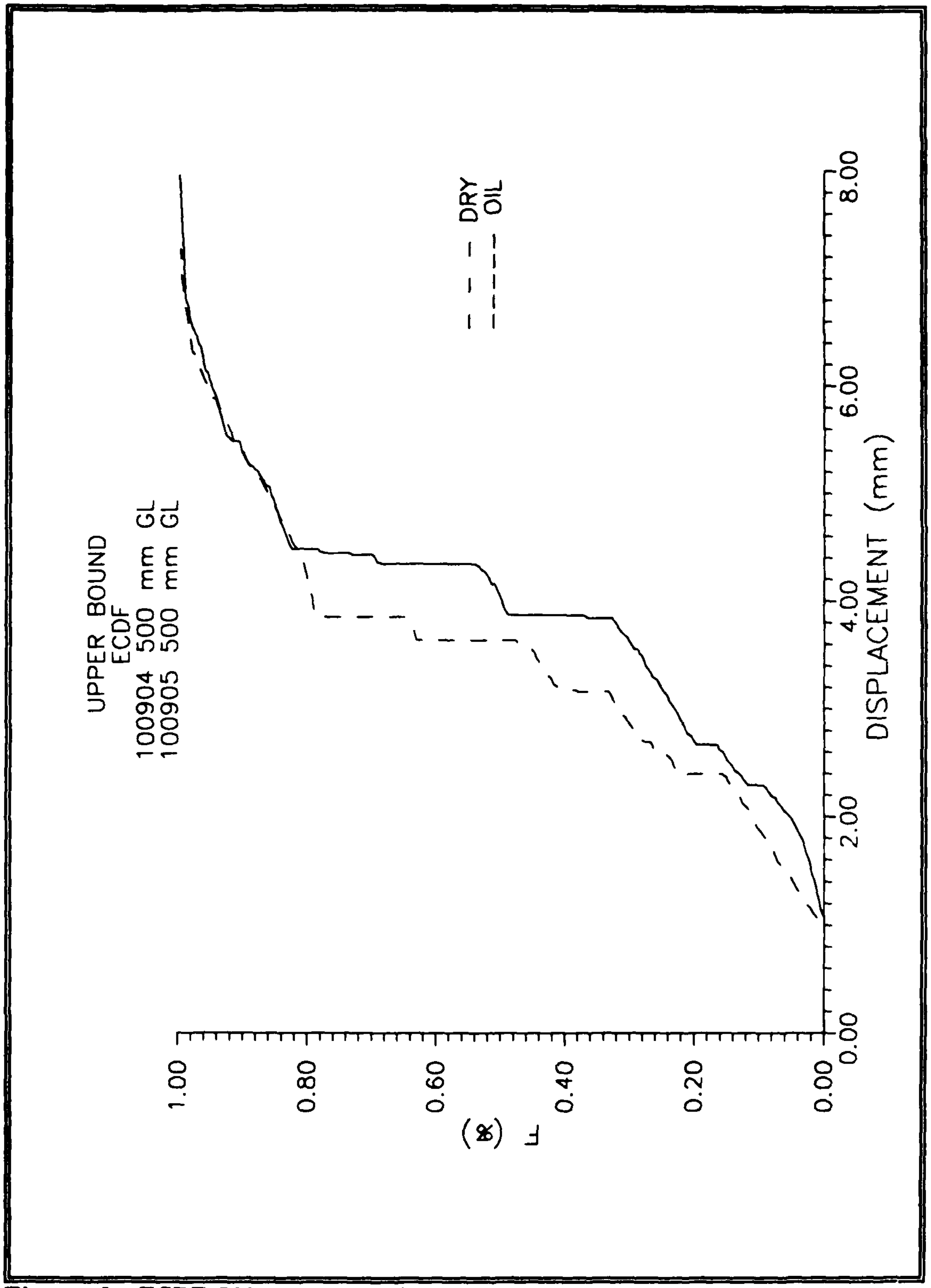

Figure 6. ECDF 500 mm comparison.

The ECDF plot (Figure 8) indicates friction had more influence than was revealed in the load curve. The upper tails again merge, indicating either the oil no longer decreases the friction or friction no longer affects the remaining fibers in this region of the ECDF. A slight change can be seen between the two samples in the lower and mid region but this could be variations between bundles and not whether the specimen was treated with oil, but it should not be dismissed because it is consistent with the other samples.

A 50 mm GL test (Figure 9) showed results which had been anticipated in the longer lengths. Little change is observed as few fibers fail but the affects of friction become prominent as the percentage increases.

The ECDF (Figure 10) indicates the effect of oil more clearly. The entire range of failure is shifted not only at the upper tail but the lower tail as well.

Determining how well the underlying Weibull distribution is extracted from a bundle test and what influence the oil treatment has can be observed in Weibull plots. For the 500 mm sample (Figure 11 & 12) friction has definitely affected the distribution, but it is also observed that oil helped to reduce some of the deviation especially in the lower tail.

As the gauge length is reduced it becomes apparent from the Weibull plots (Figures 13,14,15 & 16) for the 250 mm GL and the 50 mm GL that friction is not necessarily negligible, for the distribution is affected. This is clearly demonstrated in the 50 mm GL treated with oil (Figure 15) which shows the characteristic Weibull distribution distinctly as apposed to the dry sample.

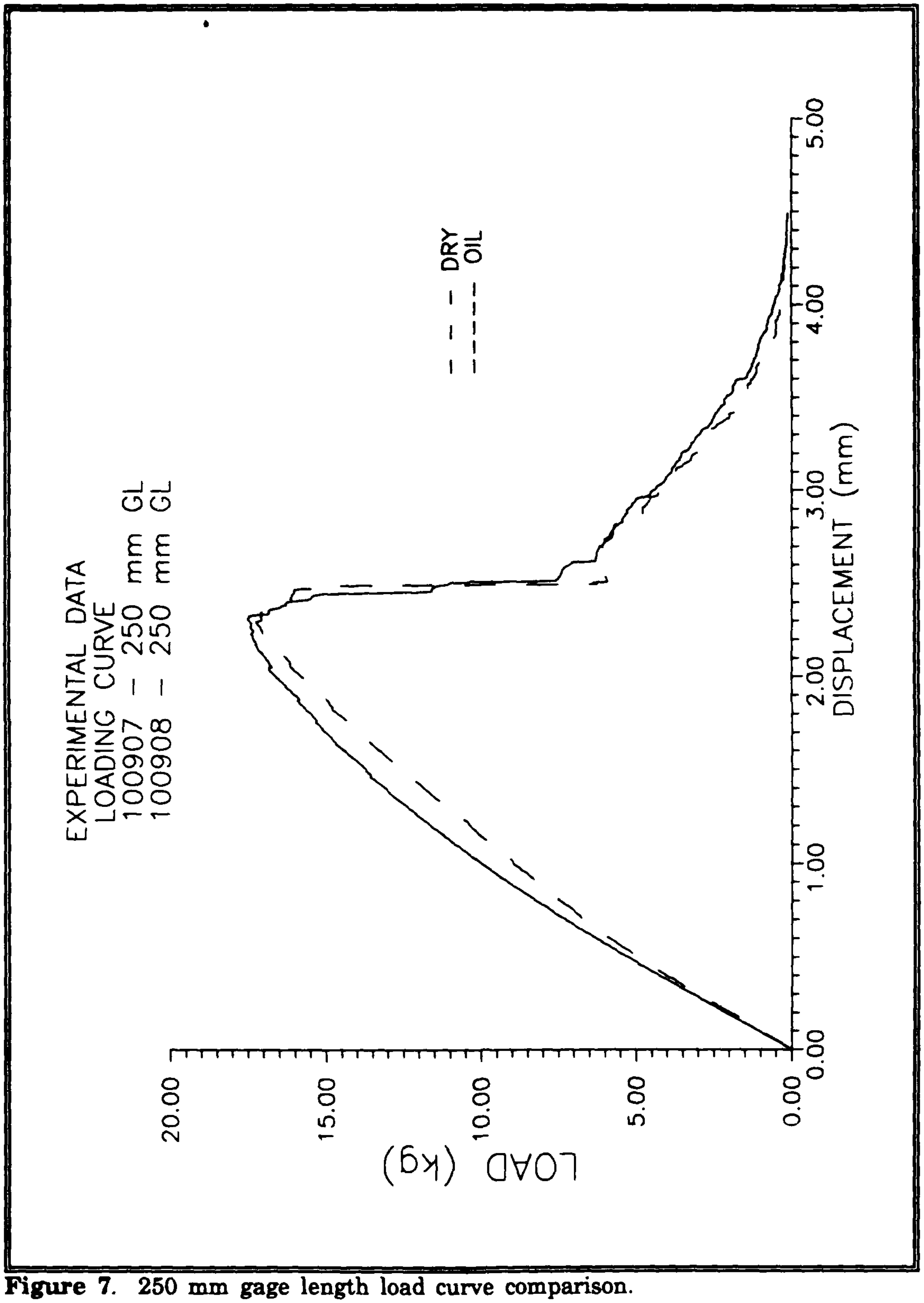

Figure 7. 250 mm gage length load curve comparison.

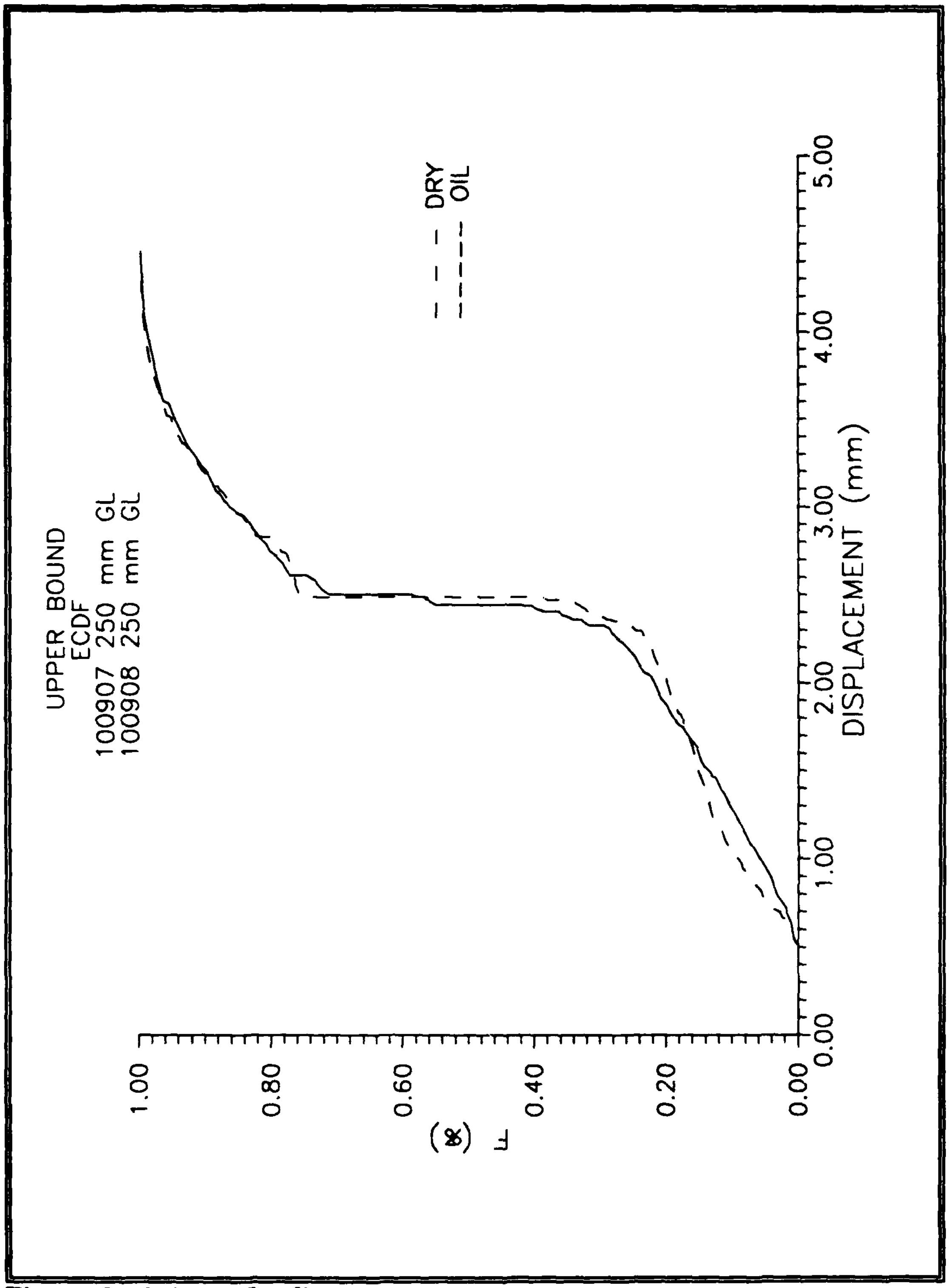

Figure 8. 250 mm Loading curve comparison.

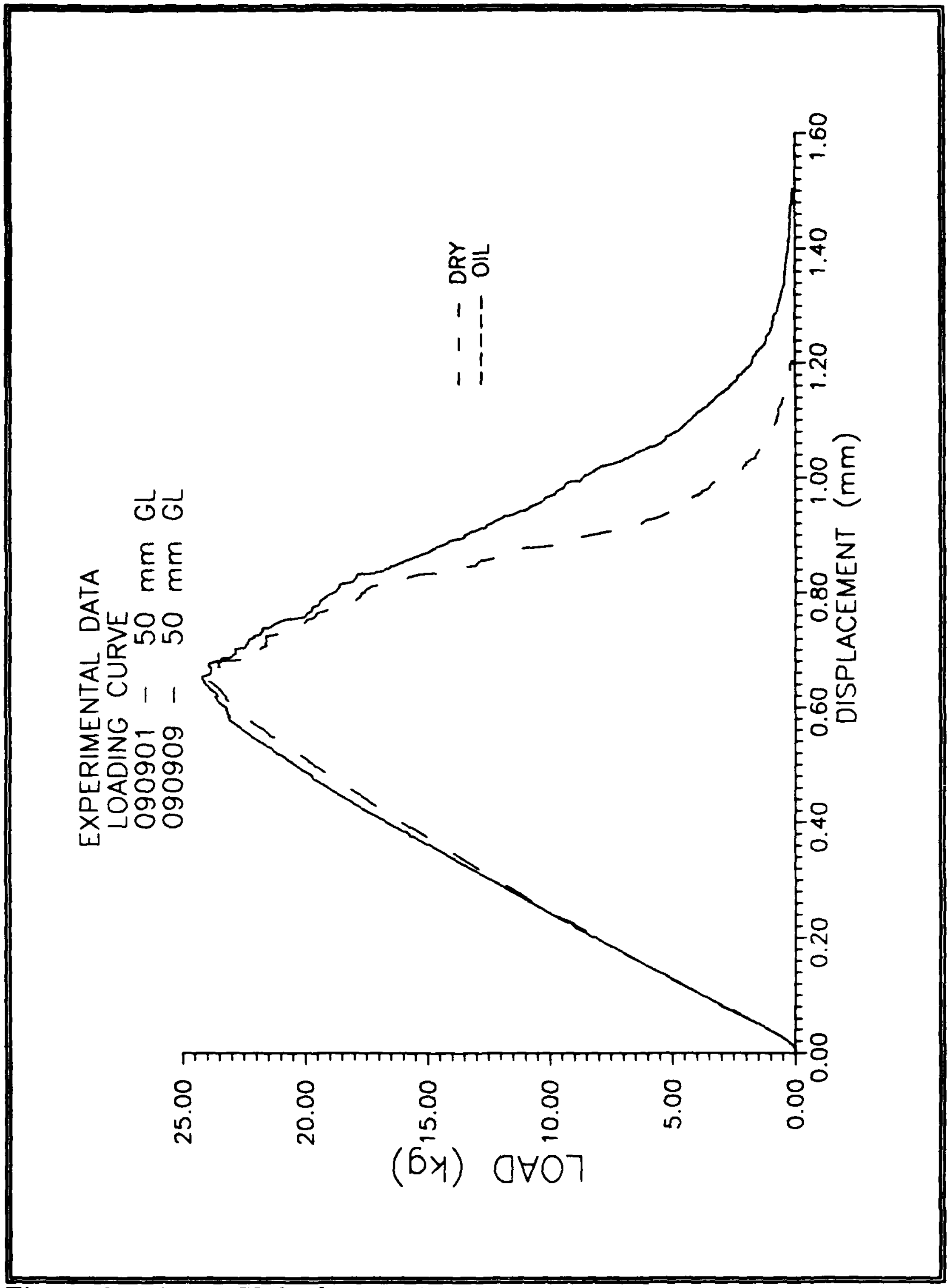

Figure 9. 50 mm GL load comparison.

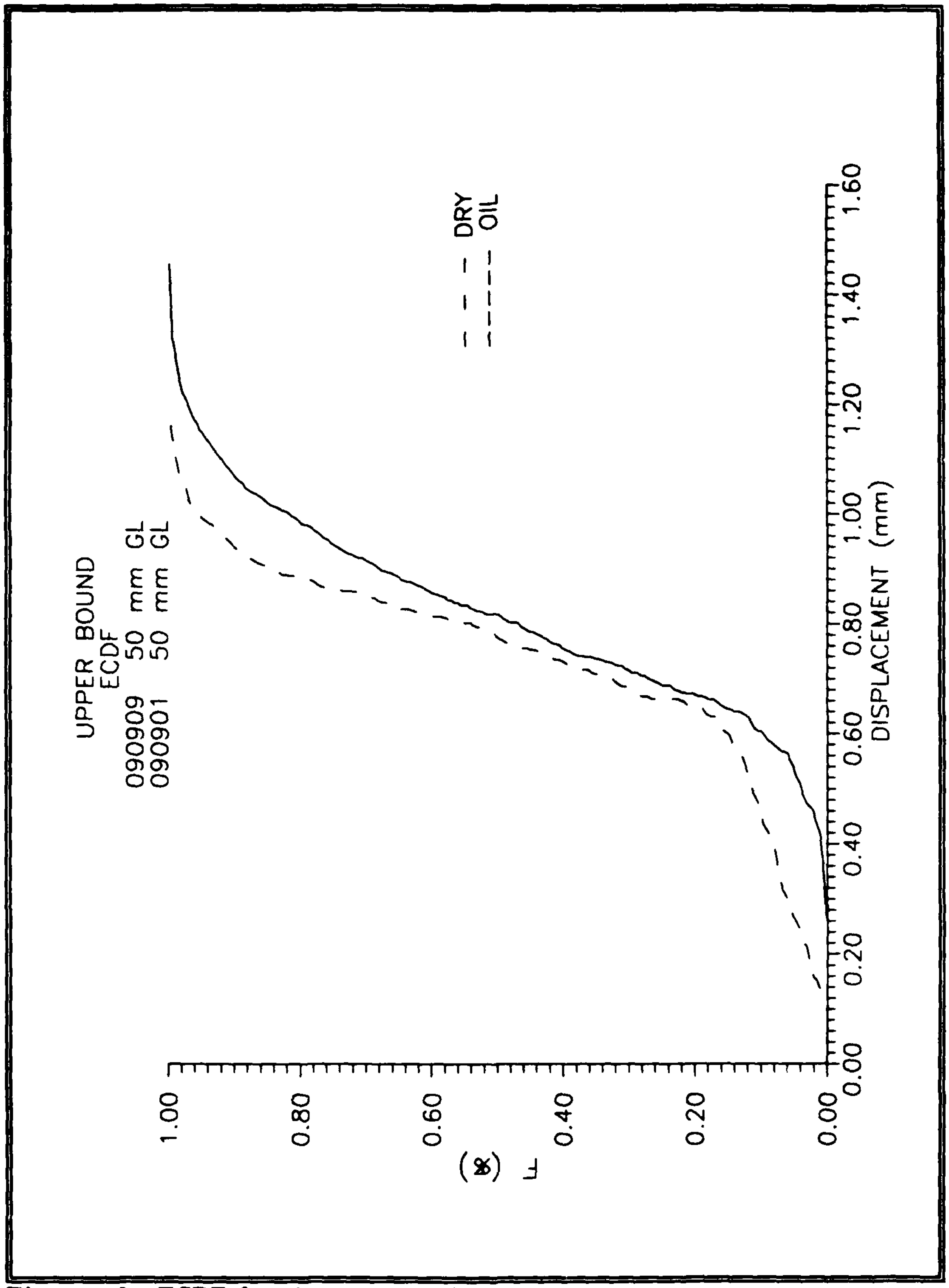

Figure 10. ECDF for 50 mm comparison.

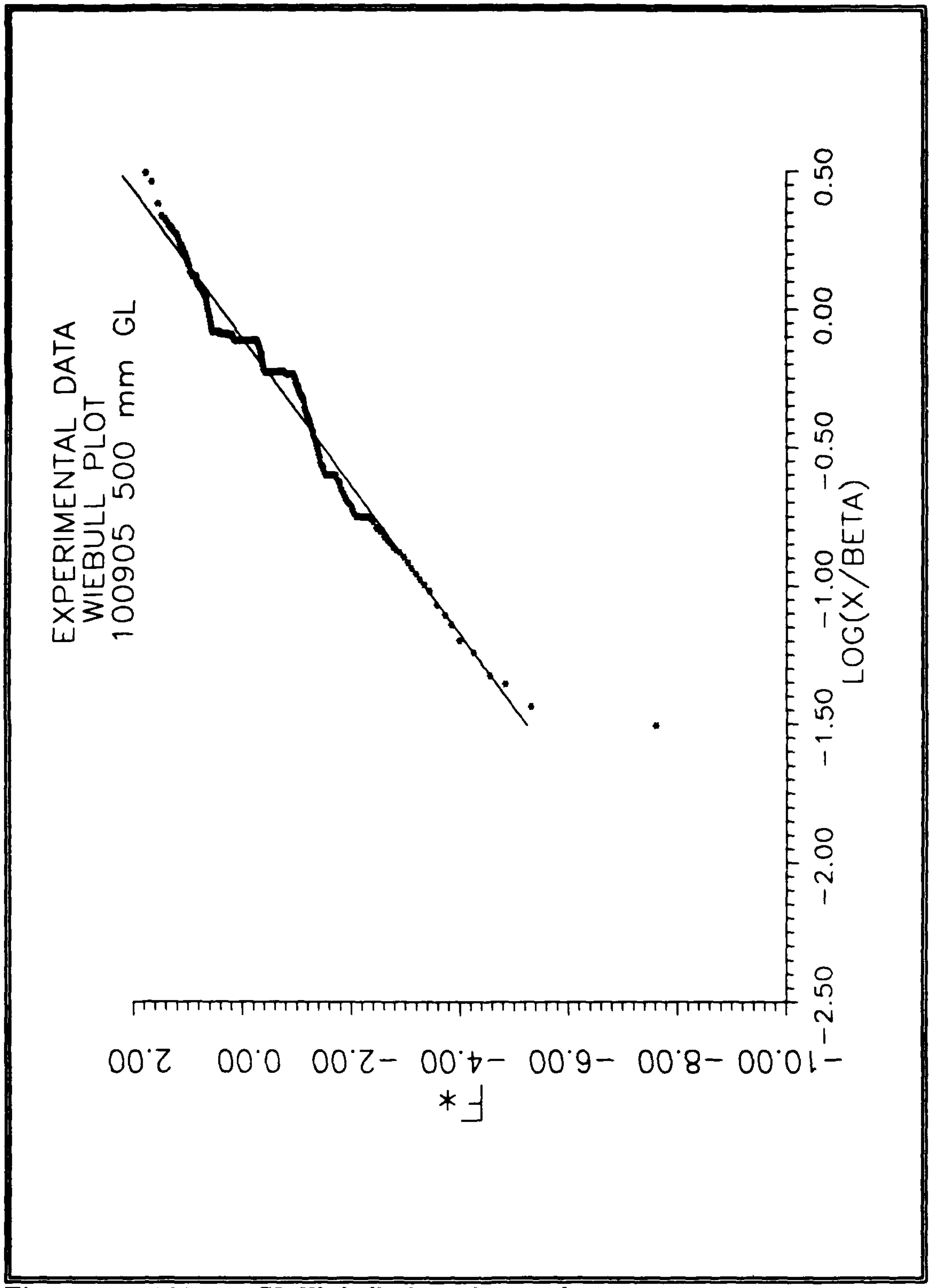

Figure 11. 500 mm GL Weibull plot, oil treated.

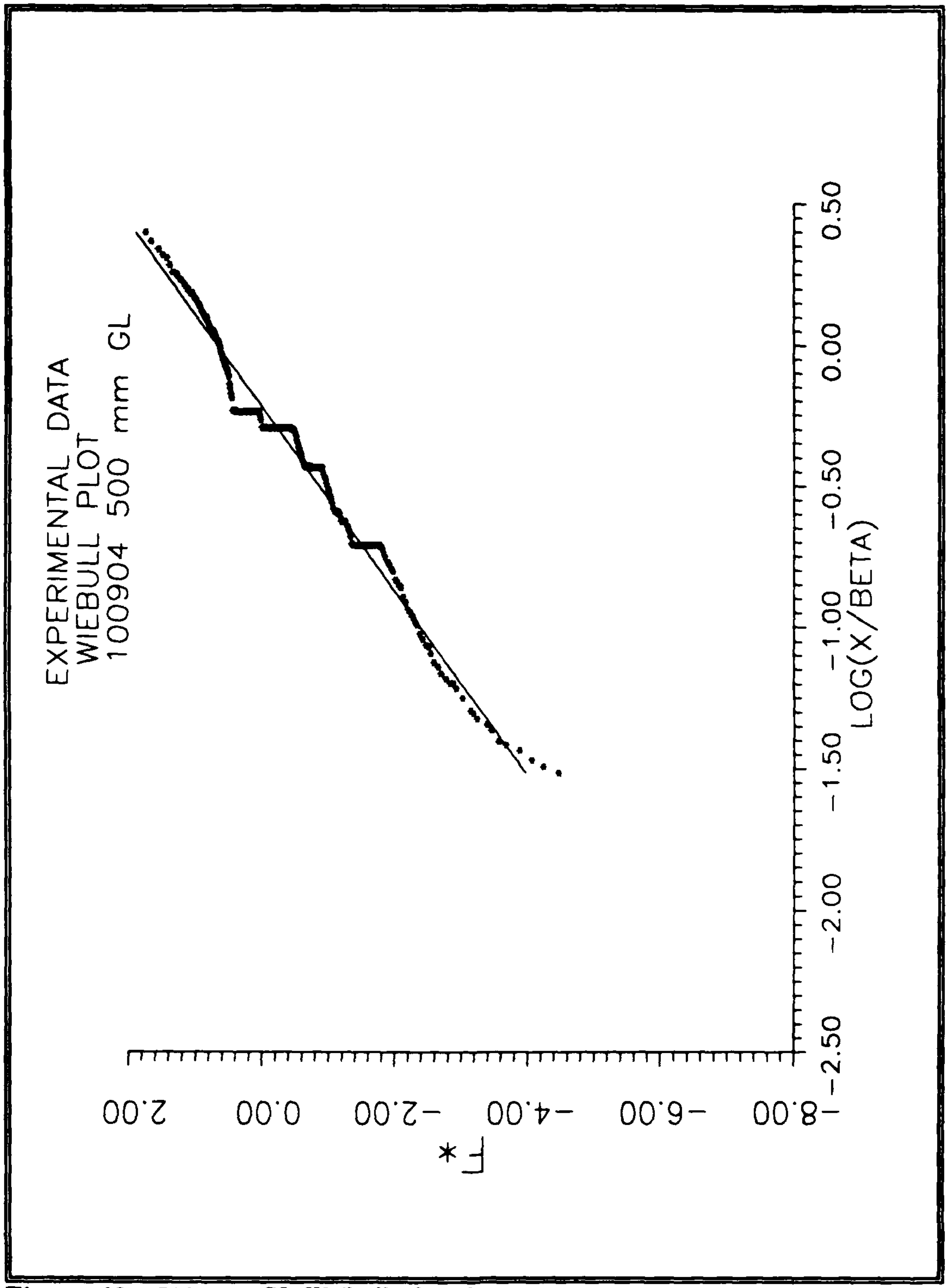

Figure 12. 500 mm GL Weibull plot, dry.

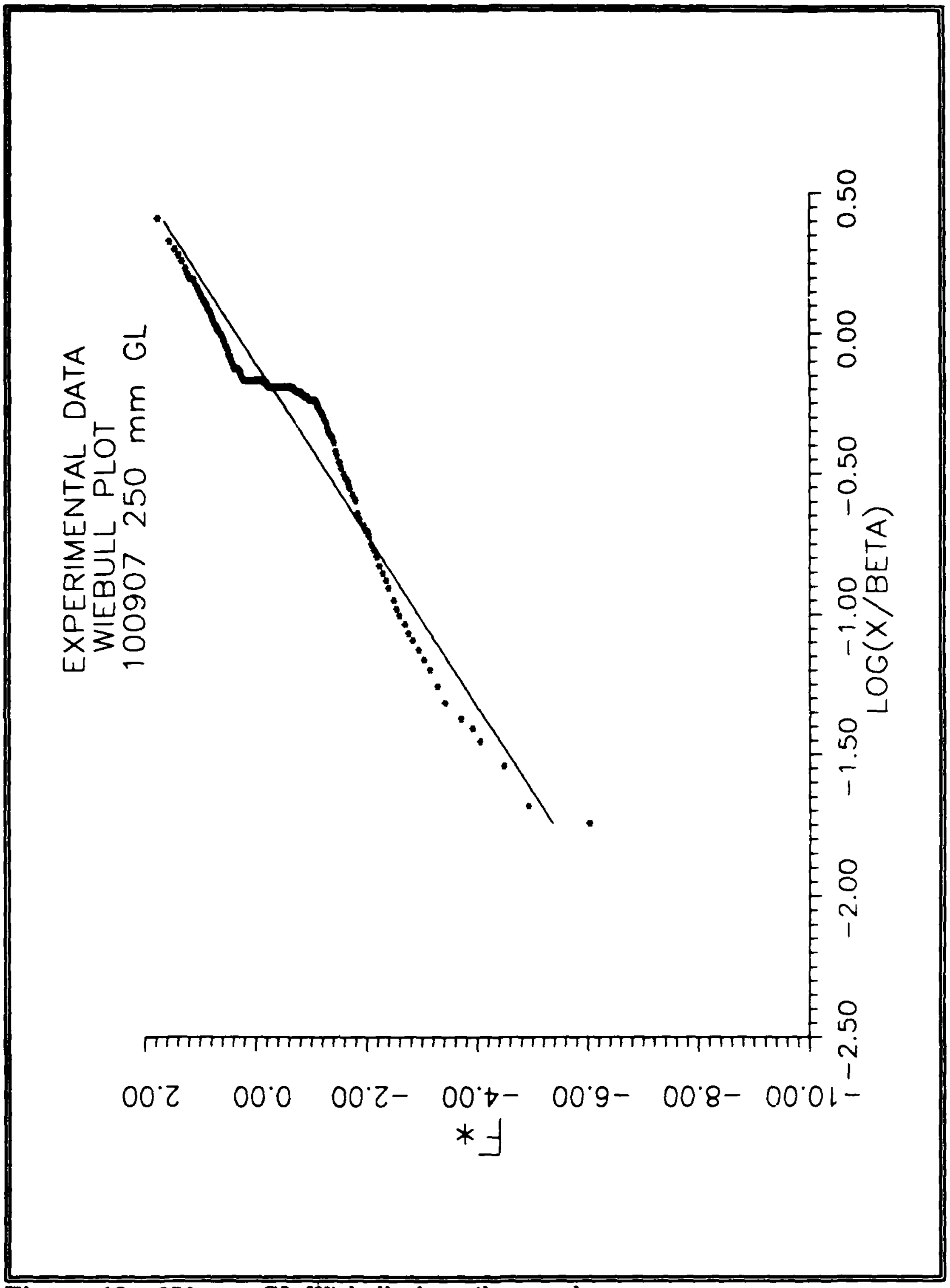

Figure 13. 250 mm GL Wiebull plot, oil treated.

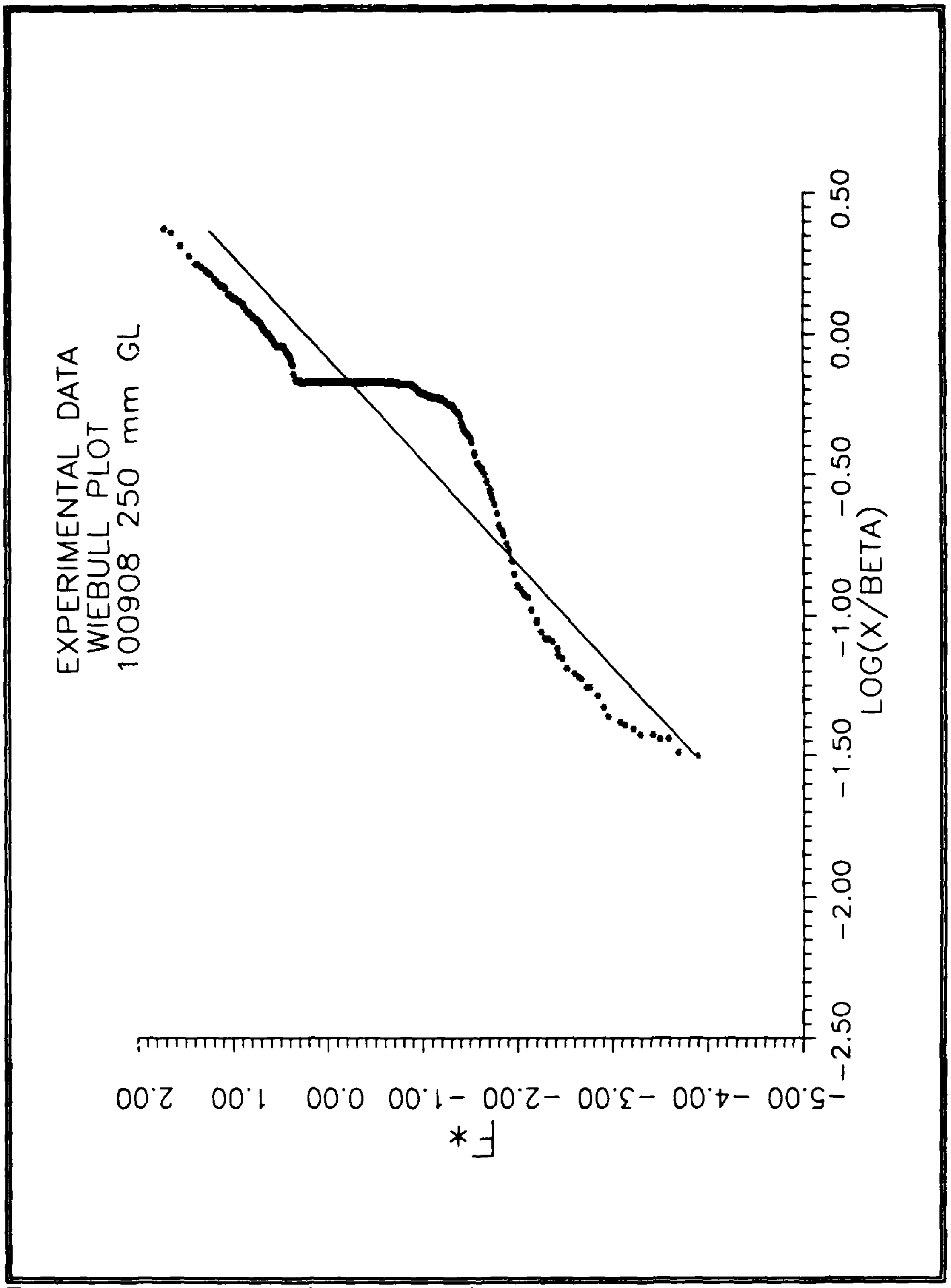

Figure 14. 250 mm GL Wiebull plot, dry.

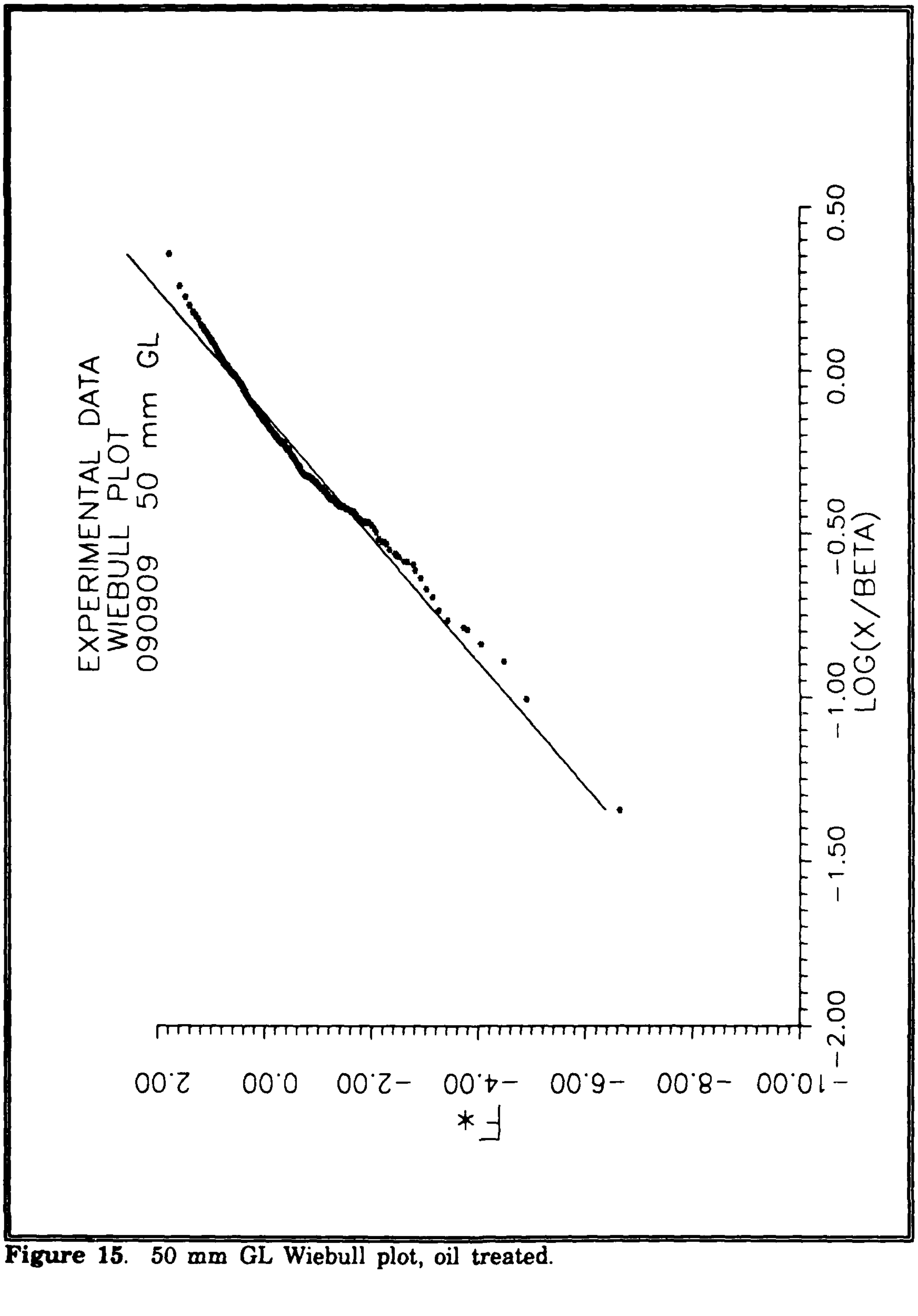

Figure 15. 50 mm GL Wiebull plot, oil treated.

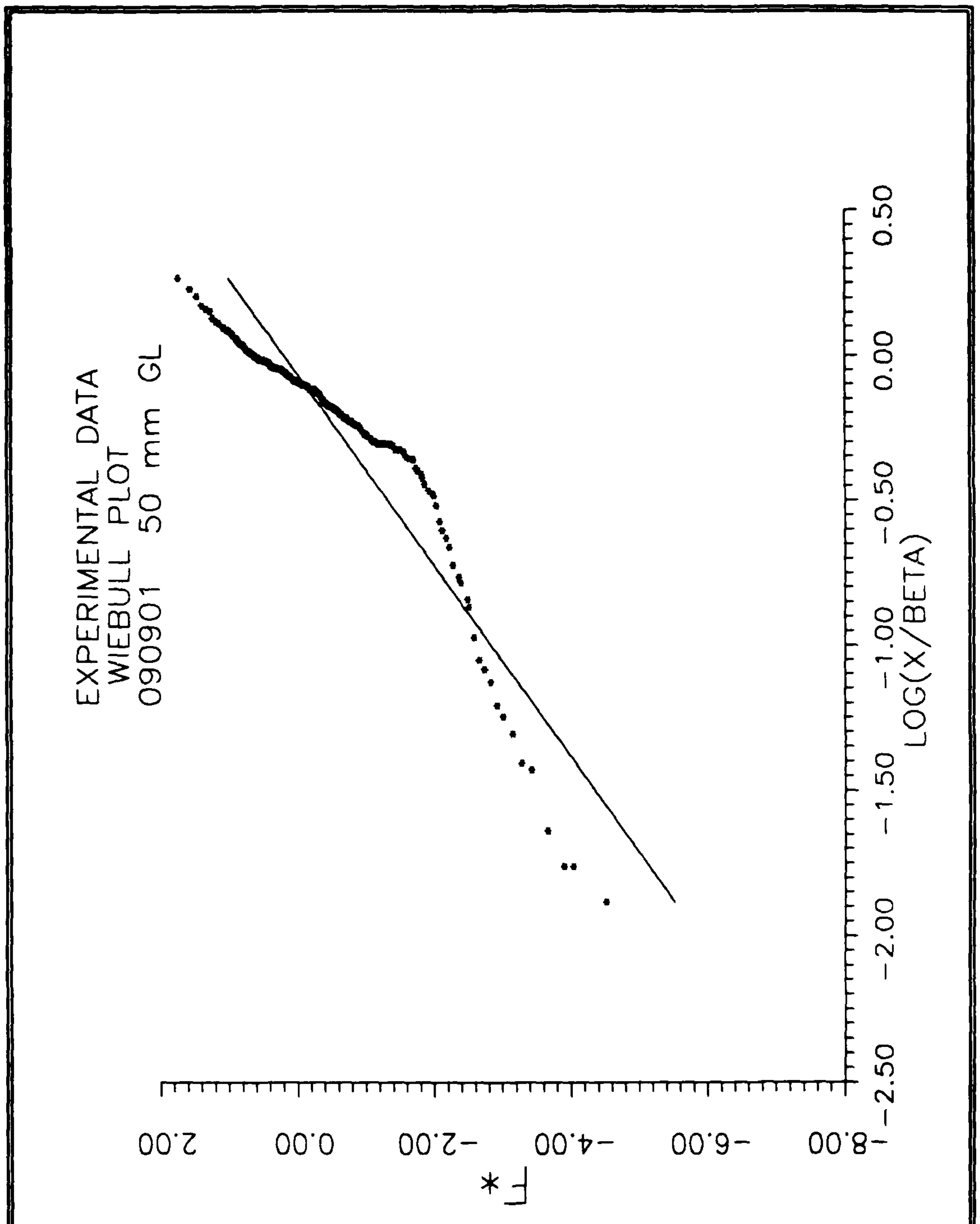

Figure 16. 50 mm GL Wiebull plot, dry.

VI. SUMMARY AND RECOMMENDATIONS

A. SUMMARY

The affect of friction is evidenced in all gauge lengths and is not always limited to the upper tail. Lubrication of the bundle reduces the effect of friction and should be used in all bundle failure testing.

The compliance removal was not used in the data reduction due to the error in the compliance curve caused by adhesive coupling failure in the tabs. The compliance ended up being over estimated. This a side the trend of compliance removal on the data reduction (Table II) indicated a general reduction in the parameters (α,β). Estimating the compliance to be much less and extrapolating this thought, it can be assumed the value for the parameters presently determined would be slightly less. In a comparison of the single fibers parameters with the bundle parameters (Table III), the bundle parameters were barely higher. Given the affect of compliance it can be concluded that once removed the bundle parameters will be indistinguishable from the single fiber testing and in fact given the quantity of the statistical base much more reliable.

It was shown that friction effected all the gauge lengths but the longer gauge length (500 mm) experienced the most disturbance. The 50 mm gauge lengths when treated with oil produced a very smooth distribution in the Weibull plot. The dry 50 mm sample produced a poor Weibull plot. The 50 mm gauge length produce good results through out the entire range. The 500 mm gauge length produced good results in the Weibull plot at the lower tail (Figure 7). Since the lower tail of the large gauge length are statistics of much

higher reliability than the 50 mm the linear statistics of the 500 mm can be pieced into the 50 mm statistical plot there by extending the range of the variable (strain). The upper tail of the 50 mm gauge length is not completely linear in the Weibull but this can be removed and statistics from a 5 mm test can replace it. This piece meal work on the Weibull plot can be done because of size effect. The slope alpha does not change but beta decreases in the Weibull as the gauge length increases. With this procedure friction and other contaminates can be removed and the underlying parameters accurately predicted.

Through simulation a normal or uniform distribution of slack was determined to have very little effect on the distribution if the d0 displacement is subtracted.

B. RECOMMENDATIONS

To ensure better results several areas of experimentation can be improved on.

- The handling of the specimens should be reduced, a rigid carrying mechanism which can transport the bundle from assembly to storage to testing without any human handling or disturbing the fibers is necessary.
- The manual mechanical grips should be replaced with a pneumatic device.
- Oil treatment should be used on all samples.
- Improve the tab coupling.

The slack distribution requires more investigation than was addressed here. Simulation was a key element which helped to determine it's effect on the distribution. An analysis algorithm is provided in this thesis which will help to determine the slack ECDF. To further study the question of slack several

bundle samples (large gauge length) dedicated solely for the determination of slack should be tested. A much larger data base is required to use the methods discussed in Appendix C. By decreasing the cross-head rate and increasing the acquisition rate a large data file can be attained. Once the distribution is fitted to a model, simulation can determine what affect it has on the bundle strength distribution.

VII. CONCLUSIONS

The results achieved in this thesis conclude convincingly that the distribution of the fiber can be retrieved through bundle failure tests. The parameters extracted from the testing compared favorably with the single fiber testing. As a result, enormous characterization efficiency is realized; in fact with this technique, statistics to 10^6 are realized.

APPENDIX A

SIZE EFFECT

The determination of the scale parameter beta of a gage length, which is too short to preclude practical experimental implementation, can be accomplished by size effect of the weibull distribution function.

Starting with the Weibull CDF for a fiber of defined gage length l_1:

$$F_1(x) = 1 - \exp[-(x/\beta_1)^{\alpha_1}] \qquad (A.1)$$

$$R_1(x) = 1 - F_1(x) \qquad (A.2)$$

For a second fiber of gage length l_2 with,

$$l_1 < l_2 \quad \text{and} \quad l_2/l_1 = m \; ; \quad m = \text{integer} > 0$$

and assuming the reliability R_t is constant.

$$R_t = (R_1)(R_2)(R_3)....(R_m) = R_1^{\,m} \qquad (A.3)$$

substituting A.2 for R_1

$$R_t = \exp[-(x/\beta_1)^{\alpha_1}]^m \qquad (A.4)$$

and the total reliability

$$R_t = \exp\{-(x/\beta_m)^{\alpha m}\} \qquad \text{(A.5)}$$

equating equation A.4 and A.5

$$\exp\{-(x/\beta_m)^{\alpha m}\} = \exp\{-(x/\beta_1)^{\alpha 1}\}^m \qquad \text{(A.6)}$$

taking the natural log A.6 reduces to

$$(x/\beta_m)^{\alpha m} = m(x/\beta_1)^{\alpha 1} \qquad \text{(A.7)}$$

again taking the natural log of A.7

$$\alpha_m \ln(x/\beta_m) - \alpha_1 \ln\{m(x/\beta_1)\} = 0 \qquad \text{(A.8)}$$

since $\alpha_m = \alpha_1 = \alpha$ is not $= 0$

$$\beta_1/\beta_2 = m^{1/\alpha} \qquad \text{(A.9)}$$

APPENDIX B

EXPERIMENTAL PROCEDURES

A. EXPERIMENTAL SET UP

The Test apparatus consisted of an INSTRON Model 4206 material tester, with a 50.0 kg load cell, connected to a 4200 Series Expanded Control Console. The Console was connected to a IBM PC/AT through com port 1 via a IEEE connection and converter. The data acquisition and material tester control was commanded by Instron Series IX Automated Material Testing System series 4.01C software.

The controlling software has a 5 page menu which requires setting prior to testing. Many of these settings are for Instron data reduction processing which are not used in this thesis but required values to operate. The settings crucial to testing and proper acquisition are cross-head speed, data acquisition rate and gage length. Safety features include a maximum load and displacement setting to insure the load cell and testing model are not damaged.

B. INSTRON CALIBRATION AND TESTING

The Instron 4206 and control unit required a warm up time of one hour prior to operation to allow for stabilization. The load cell has two methods for calibration, an electronic calibration and a mechanical calibration. A mechanical calibration was performed for all tests.

1. Mechanical Calibration

Any inputs refer to the 4200 Series Expanded Control console.

- Main power on/off/on - wait for diagnostic

- Press Load Balance / Enter
- Hand prescribe a 5.0 kg weight
- Press Load Cal
- Enter the weight / Enter
- Remove the weight
- Enter Load Balance / Enter
- Re-hang the weight to verify correct calibration
- Repeat if required

2. Loading the Sample

The procedures for sample placement into the test apparatus are as follows.

- The Instron specimen grips are separated enough to allow the sample to hang from the top grip without touching bottom grip. The tab of the specimen is firmly clasped with tweezers and gently lifted off resting bench allowing to hang free.

- The samples top tab is placed in the top grip, careful to align the bundle with the center, which has been measured and marked. After alignment the grip is tightened making sure not to twist or damage the specimen.

- A load balance is performed at the control panel to remove the weight of the specimen.

- The cross-head is then toggle down so as to place the bottom tab in the bottom grip then tighten. Note: after tightening the grips the specimen develops noticeable slack and the load cell measures a negative load. The specimen fibers are slightly compressed and need to be straightened.

- The cross-head is toggle up until a load of 0.1 kg is indicated in the load readout window on the control panel. The load is then brought down to zero and the displacement reference zeroized. The procedure is repeated once to ensure the displacement returns to at or near zero. The specimen is ready for testing.

3. Testing

The test is initiated at the IBM PC/AT through the Instron software. After the specimen test parameters are set and loaded, prior to test initiation,

the IEEE port is enabled. The software then signals the system is ready to begin testing.

As the cross-head displaces, load and displacement data are sent to the PC/AT at regular intervals of displacement, as prescribed by the initial software settings.

A real time graphical output is available to monitor the testing. This allowed for a quick identification of bad tests. After the test was complete, the data file is converted from Instron System code to ASCII for latter data analysis.

C. SAMPLE PREPARATION

The samples were made from a Hercules Magnamite high strength graphite, type AS-4, spool 145. The bundle has 3000 fibers with a denier of .0057446 gm/in..

1. Procedures

A length of bundle is clamped at one end on a 3 meter aluminum track and weighted on the other. Half Copper tabs approximately 2.54 cm (width) x 2.54 cm (length) are pre-positioned on the track at a specified gage length. Slots built into the track, center the bundle. An adhesive is applied at the tab, the bundle and the remaining half of the copper tab are securely clamped on to complete the tab. The bundle is severed between samples on the track and the adhesive is allowed to cure.

Once dry the samples are manually removed from the track and placed on the sample bench.

D. CALCULATION OF CROSS-HEAD SPEED

The cross-head calculation is a function of the number of data points required, data acquisition rate, load cell and displacement gage tolerance.

The time to bundle failure was selected to remain constant. This will provide a constant strain rate on the different gage lengths that were tested.

The relationship between the cross-head, acquisition rate and recorded range is given in equation B.1.

$$XH = (RR)(AR)/DPTS \tag{B.1}$$

where: XH = cross-head speed
RR = recorded range
AR = acquisition rate
DPTS = number of data points recorded

The sampling interval is

$$deltaD = RR \ / \ DPTS = XH \ / \ AR \tag{B.2}$$

In order not to sample beyond the abilities of the acquisition system the following rule must be followed

$$deltaD \geq \text{Recording Tolerance} \tag{B.3}$$

The Instron output tolerance for displacement was 0.00254 mm, therefore deltaD ≥ 0.00254 mm. Using this as a guideline and the constant testing time chosen of 5 min.. Table IB lists the Instron test setting calculations.

Table IB. TEST SETTINGS.

Gage length (mm)	Max displ. (mm)	Time (min)	Cross-head (mm/s)	Acq. rate (pts/s)
500	25	5	8.33×10^{-2}	6.6
250	12.5	5	4.17×10^{-2}	3.3
50	2.5	5	8.33×10^{-3}	3.3
25	1.25	5	4.17×10^{-3}	1.67

APPENDIX C

DATA ANALYSIS

The analysis of the bundle failure load vs displacement curve can be divided into three distinct regions (see Figure 1C):

- Slack region.
- Linear region.
- Non-linear region.

Slack is created during specimen production and placement in the Material testing machine. As the bundle is measured, cut and tabs are placed on each

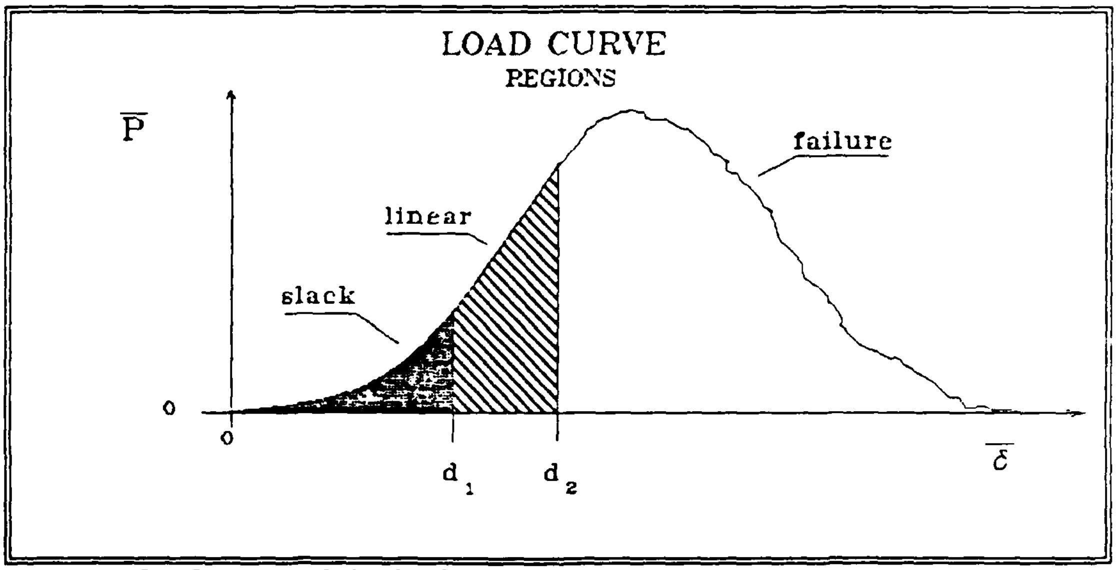

Figure 1C. Regions of the load curve.

end of the sample some of the individual fibers become loose and effectively longer than other fibers. This produces an uneven loading across the fibers as the cross-head of the tensile testing machine strains the bundle. The i^{th} fiber is

not loaded until the displacement of the cross-head is equal to the respective slack of the i^{th} fiber.

When the cross-head displaces beyond the maximum slack, the loading curve becomes linear until the first failure.

The amount of slack varies according to an undefined distribution. The slack distorts the failure region thereby disturbing the underlying sequence of the strength distribution. The effect slack has on the underlying shape parameter depends on the slack distribution and amount of slack relative to the total strain, this is demonstrated in Appendix D.

A. SLACK DISTRIBUTION

The slack in a bundle is dependent on the variation in the fiber lengths.

$$L_j = l + delL_j \tag{C.1}$$

where: Lj = length of fiber j
l = mean gage length
$delL_j$ = slack of fiber j

Since the $delL_j \ll l$ the length is approximated by

$$L_j \approx l \tag{C.2}$$

The load in the slack region can be defined by Hooke's law.

$$P_S(d_A)_k = \sum_{j=1}^{k-1} E\ [(d_A - delL_j)/(l + delL_j)] \tag{C.3}$$

$$\approx \sum_{j=1}^{k-1} E\ d_j/l \qquad ; \qquad k=1,2,....,n$$

where: $P_s(\ )_k$ = bundle load as each fiber k is loaded
E = fiber modulus (gm/strain)
d_A = apparent displacement = $d_j + delL_j$
d_j = fiber displacement
n = number of fibers

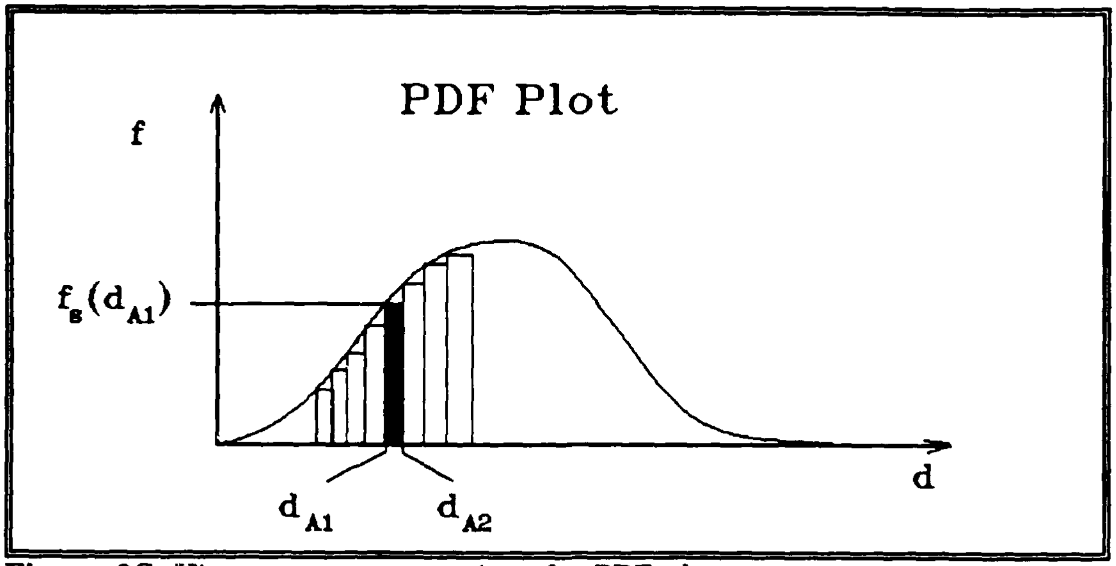

Figure 2C. Histogram representation of a PDF plot.

Defining the loading slack in terms of the distribution function (f_S) of P_S at a displacement d_A as shown in Figure 2C. P_s can be write as

$$P_S(d)_1 = \{f_S(d_{A1})(d_{A2}-d_{A1})n\} \; E \; U(d-d_{A1})\{(d-d_{A1})/l\} \qquad \text{(C.4)}$$

where: d = any displacement such that $d > d_{A1}$

$U(d)$ = unit step function $\{=1 \; d \geq d_{A1};$ $\{=0 \; d < d_{A1}$

Summing the load (eqn. C.4) over the range of d_A and defining the bundle modulus as a function of the fiber modulus

$$\underline{E} = nE/l \qquad \text{(C.5)}$$

$$P_S(d) = \underline{E} \; \Sigma \; [f_S(d_{A\,i})(d_{A\,i+1} - d_{A\,i})] \; U(d-d_{A\,i}) \qquad \text{(C.6)}$$

differentiating with respect to d for a continuous function

$$\delta P/\delta(d) = E \int_{d_1}^{d_i} f_s(d)\, d(d) = \underline{E}\ F(d_i) \qquad \text{(C.7)}$$

$F(d_i)$ is the CDF of the slack. Assuming the empirical data approximates the continuous data.

$$P' = F \approx \underline{F} = \underline{P}' \qquad \text{(C.8)}$$

The derivative of the empirical data can be numerically solved with two methods, a numerical differentiation or a discretization.

1. Numerical Differentiation

Even interval Forward and Central difference methods are used due to the nature of the slack loading and the recording system. As the fibers load the slope is discontinuous as seen in Figure 3C. Several numerical methods exist which will produce varying accuracy. In trying to recover the underlying distribution of the slack the sampling rate and number of fibers must be considered (i.e., how many data points can be recorded relative to the number of fibers).

If the sampling rate is fast and the number of total data points i recorded in the slack region is 2-3 times that of the fiber, a 3 point

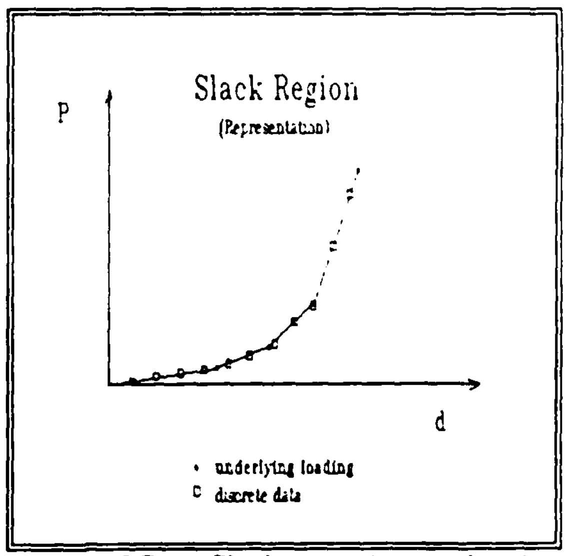

Figure 3C. Slack region showing discrete data on the underlying loading slope.

forward difference method may yield accurate results due to the linearity between discrete data. If the relative sampling rate is slow the 3 Pt. forward difference may skew the distribution. To prevent this a central difference method is safe.

3 pt. Central Difference method.

$$\underline{P}_i' = (-\underline{P}_{i-1} + \underline{P}_{i+1})/2h \tag{C.9.1}$$

5 pt. Central Difference method.

$$\underline{P}_i' = (\underline{P}_{i-2} - 8\underline{P}_{i-1} - 8\underline{P}_{i+1} - \underline{P}_{i+2})/2h \tag{C.9.2}$$

where: $h = \underline{d}_{Ai+1} - \underline{d}_{Ai}$

2. Discretization

The data in the slack region can be fit to a curve and a equation estimated. This equation can be differentiated analytically. The underlying data itself is not smooth so such a method is not recommended.

B. LINEAR REGION

The slope of the linear region is the bundle modulus. To calculate this slope a least squares method is very accurate, due in part to the natural linearity of the data. Before this method can be employed the region must be specifically defined. The point where the non-linear slack region ends is identified as d1 and the end of the linear region is identified as d2 (the point of

the first failure). This is graphically displayed in Figure 4C. Between d1 and d2 the least squares method calculates modulus $\underline{E}$.

The points d1 and d2 must be graphically identified and manually entered into the analysis program. To do this a plot of the load curve must be generated before the analysis can be conducted.

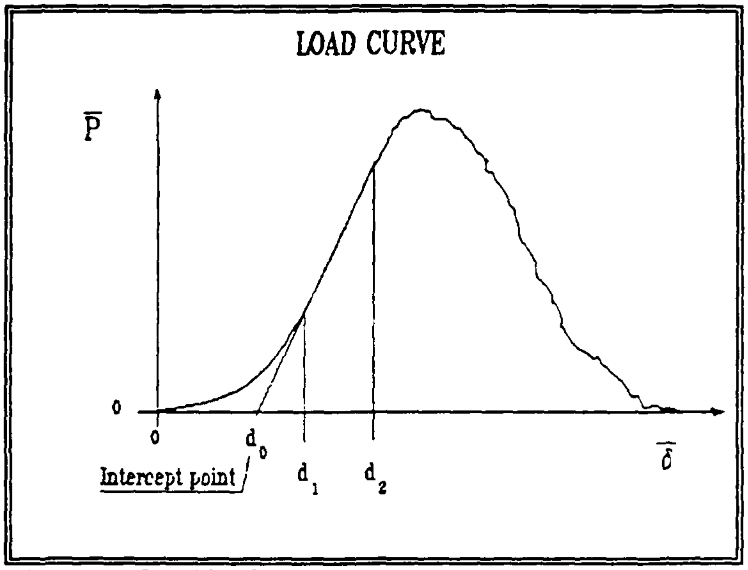

Figure 4C. The linear region.

1. Least Squares Method

Due to the noise, the discrete data i will deviate slightly from the underlying slope $\underline{E}$. From Hooke's law the expected relationship is

$$\underline{P}_i = \; = \underline{E}D_i \tag{C.10}$$

where: $D_i \; = \underline{d}_{Ai}\text{-}\underline{d}1$ (C.10.1)

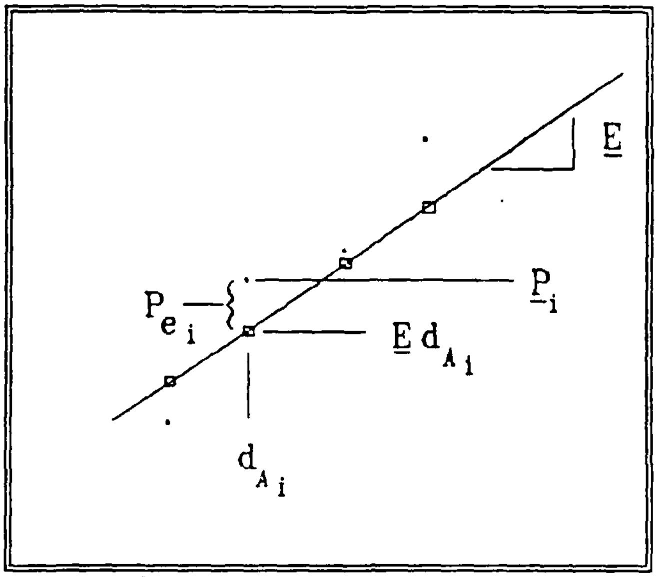

Figure 5C. Least Squares Method.

But due to noise the load $\underline{P}$ deviates by an amount Pe, this is graphically represented in Figure 5.

$$Pe_i = \underline{E}D_i - \underline{P}_i \tag{C.11}$$

Defining the least squares sum M

$$M(\underline{E}) = \Sigma \{(\underline{E}D_i - \underline{P}_i)/2\sigma^2\}^2 \tag{C.12}$$

where: σ = standard deviation

Differentiating M with respect to $\underline{E}$ and setting it equal zero defines the minimum least squares.

$$dM/d\underline{E} = 0$$

substituting in C.10.1 this reduces to

$$\underline{E} = \Sigma \underline{P}_i(\underline{d}_{Ai}-\underline{d}1)/\Sigma(\underline{d}_{Ai}-\underline{d}1)^2 \qquad \text{(C.13)}$$

2. Intercept of the Abscissas

With the Modulus $\underline{E}$ the intercept of the abscissas can be calculated.

$$d0 = d1 - \underline{P}(d1)/\underline{E} \qquad \text{(C.14)}$$

C. FAILURE REGION

This region is the key to retrieving the strength distribution. Using the slope of the bundle the failure statistics can be transferred to a ECDF space. From the ECDF the parameters of the distribution can be estimated.

The ECDF is a plot of the percentage of failures versus a variable. Typically this variable is the strength, this would be difficult to translate due to the nature of the bundle test. The displacement is constantly incremented, where as the load is a bundle load and can vary. Because of this the ECDF is plotted versus the displacement. The displacement is proportional to the strength as defined by Hooke's law and once the fiber modulus is identified the displacement can be translated to load.

When a fiber fails, the bundle modulus $\underline{E}$ changes by one fiber strength. From Hooke's law it is expected that the failures would occur along the modulus E_j. E_j being the modulus of the unfailed bundle which ideally is a multiple of the single fiber modulus proportional to the number of remaining fibers. Unfortunately this is not the case, as the bundle loads, friction and slack change the distribution causing the actual loading path to deviate from E_j and follow along $\underline{E}_i$.

To show this in more detail consider first the ideal underlying loading (no slack, friction or noise) of a bundle under constant strain. At a break in one fiber j, the load will drop until intercepting the slope of E_{j-1} shown in Figure 6C. This new slope emanates radially out from the intercept of the abscissas, (note: without slack the intercept should be the origin). If the data acquisition rate where high enough this failure could be clearly identified. Now add slack, this extends the failure displacement of some fibers and the intercept of the abscissas by E_j after the initial failure to some point between the origin and the d0 point. The intercept will shift a relatively small amount but will digress toward the origin. If friction is present the broken fibers will cling to the unbroken fibers and cause additional strain. Add noise to the acquisition system and the discrete data fluctuates. These factors combine to force the actual recorded data to deviate from the underlying bundle loading path.

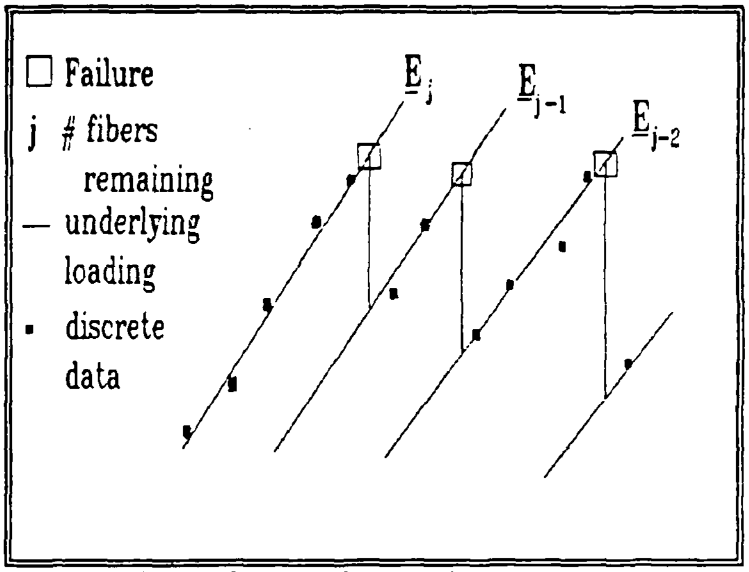

Figure 6C. Failure loading path.

A bundle with n number of fibers will have n failures. If the data acquisition system can not record this many points in the failure displacement range than each statistic can not be recorded. With slack, friction and noise

even with a fast enough data acquisition system the failures might not be distinguishable.

The percentage of failures in the bundle test can be defined by the ratio of the modulus of no failure to the modulus of failures. The reliability is defined by

$$\underline{R}_i = E_i/\underline{E} = \underline{P}(\underline{d}_{Ai})/[\underline{E}(d_{Ai}-d0)] \qquad (C.15)$$

The percent failure is 1 minus reliability, the ECDF is defined by

$$\underline{F}(\underline{d}_{Ai}) = 1 - \underline{R}_i \qquad (C.16)$$

This relationship is graphically depicted in Figure 6C. The ECDF begins at

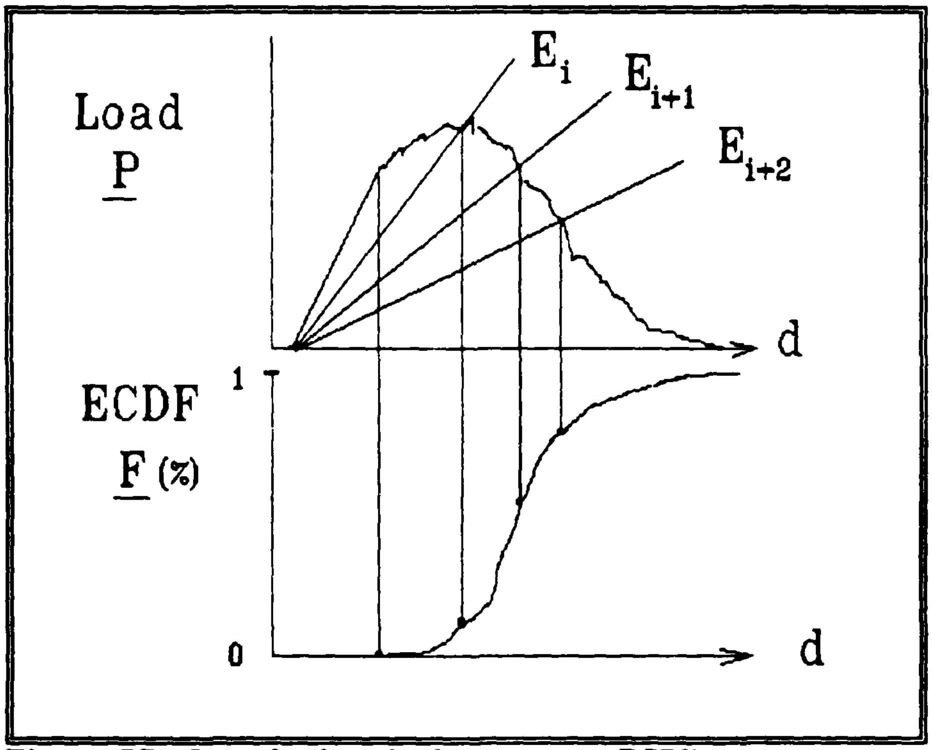

Figure 7C. Transfer from loading curve to ECDF space.

d2, so the manual selection of this point must be done with care.

Assuming all conditions ideal the ECDF would fall out as

$$F = j/n \tag{C.17}$$

But because conditions are not ideal this does not occur. What is important is the shape of the ECDF and whether it is close to the underlying CDF. The ability to distinguish each failure is unimportant. Another point to note is the lower tail needs to be clearly defined. Due to the relatively low failure rate in the lower tail and a constant recording rate this area of the curve has a high number of discrete data points per failure, so in essence the testing method takes care of itself.

1. Upper Bound

The data is recorded on even intervals of displacement not just at the failure points, this is graphically described in Figure 6. The discrete data most closely resembling the failure point is on the top edge of the loading curve. This is referred to as the Upper bound. When this is translated into the ECDF space the Upper bound is on the far edge of the plot as seen in Figure 8C. To retrieve the underlying CDF from the ECDF the Upper bound (Fu) must be extracted.

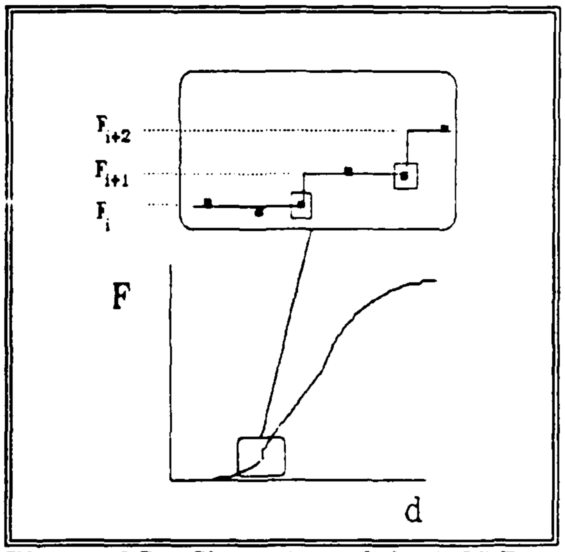

Figure 8C. Close view of the ECDF.

Attempting to extract the Upper bound from a bundle of n fibers the most failure points that could be optimistically retrieved is n. With a large n this is not possible due to the acquisition rate, no

matter, the shape and location is important and does not require a large number of points.

Using the empirical ranking as a reference point

$$F = k/m \; ; \quad k = 1,2,\ldots\ldots,m \tag{C.18}$$

where m is the number of points desired in the ECDF. F is used to define search bands of data where the upper and lower band is defined by 1/n. The value of $\underline{F}_i$ which has the largest displacement ($\underline{d}_{Ai}$) is assigned to $\underline{Fu}_k$. When no discrete data is in the search band F, $\underline{Fu}_k$ is assigned the value of F at a displacement $d_{A\,k-1}$.

2. Maximum Likelihood Estimator.

With the Upper bound extracted this data can be used to estimate the parameters of the distribution. The model selected is a Two Parameter Weibull Distribution. The estimator chosen is the Maximum Likelihood Estimator (MLE) [Ref. #:p. 103] developed by R. A. Fisher. T' ' method finds the maximum of the likelihood of the sample in the range of the variable $\underline{du}$ and is a function of distribution parameters. For the Weibull model the method solves for the two parameters alpha and beta. The parameter beta is solved explicitly [Ref. 4] in terms of alpha. Alpha can not be solved explicitly and an iteration must be used.

$$\beta = [\ 1/m\ \Sigma\ \underline{du}_i^2\ ]^{1/2} \qquad \text{sum i=1 to m} \tag{C.19}$$

$$\alpha = \{\ \Sigma\ \underline{du}_i^2\ \ln(\underline{du}_i)/(\Sigma\ \underline{du}_i^2) - \Sigma\ \ln(\underline{du}_i)\}^{-1} \tag{C.20}$$

An Atkinson method of iteration was chosen, initially alpha is assumed. From this successive alpha's are calculated using equation C.20.

$$\alpha'_o = \alpha_o - (\alpha_o - \alpha_1)^2/(\alpha_o + 2\alpha_1 + \alpha_2) \qquad \text{(C.21)}$$

where: α_o = initial start value

$\alpha_1 = f_\alpha(\alpha_o)$
$\alpha_2 = f_\alpha(\alpha_1)$

This equation is iterated for α'_o until $\alpha'_o \approx \alpha_o$.

1. Linearized ECDF

The data is linearized for a Weibull probability plot to visually check the conformity to this model. To linearize the ECDF start with the Weibull equation

$$F = 1 - \exp(-P/\beta)^\alpha$$

subtracting 1, multiplying by -1 and taking the natural log.

$$\ln(1-F) = (-P/\beta)^\alpha$$

again taking the natural log

$$\ln[-\ln(1-F)] = \alpha \ln(P/\beta) \qquad \text{(C.22)}$$

C.22 is a linear line with a slope of alpha.

D. DATA OUTPUT

As the analysis program calculates certain data file are output, these files are:

- EXPER.OUT, the converted experimental output displacement, strain and load.
- SLACK.OUT, the slack region.
- EXMS.OUT, the load curve minus slack region.

- UBFECDF.OUT, the Upper bound ECDF.
- FECDF.OUT, the ECDF.
- SECDF.OUT, slack ECDF.

APPENDIX D

A. SIMULATION

The simulation of a bundle test has two purposes. First by analyzing in depth the mechanisms which govern the failure of the fibers in a bundle, a better understanding of how to extract the characteristic properties is attained. The properties desired are the distribution parameters.

Secondly a simulation gives us the ability to know what the underlying parameters are and thereby proof the analysis data reduction algorithm. Also the variables which influence the shape of the failure distribution can be varied in simulation and the degree of induced affect observed.

The bundle loading curve can be divided into three regions;

- Slack
- Linear
- Failure

Initially the simulation requires input of the variable estimates, this is done through an input file.

1. Initial Inputs

The underlying parameters define the strength distribution for a gage length. The distribution chosen is the Weibull model. The parameters are alpha and beta, the number of fibers per bundle is n. The fiber modulus (E) is in gm/mm^2, from Hooke's law

$$\sigma = E\,\varepsilon \tag{D.1}$$

where: σ = stress
ε = strain

Substituting the definition of stress and strain for fiber i

$$P_i/A = E\ \delta_i/L_i \tag{D.2}$$

where:
P = load
A = area = constant
L = length of fiber
δ = change in length L

rearranging D.2 and letting $E_f = EA$ (gm/ε)

$$P_i = E_f\ \delta_i/L_i \tag{D.3}$$

2. Slack and Strength Distributions

The slack is caused by the variation in fiber length L. If L were constant no slack would be evident. The slack is expected to have a definable distribution, but this is unknown and could vary from bundle to bundle therefore several models are simulated. Two models selected where

- Two parameter Weibull
- Uniform.

Although physical consideration favors the slack model to be normal, the Weibull model was chosen for it's analytical convenience. The Weibull is very flexible and can characterize a wide range of shapes. For an alpha of 3.5 the distribution is approximately normal, other than 3.5 the shape of distribution is skewed negative or positive. Beta identifies the central tendency of the distribution, its relatable to the mean. From equation 2.1 the Weibull CDF is defined as

$$F_w(x;\alpha,\beta) = 1 - \exp[-(x/\beta)^\alpha$$

rearranging in terms of the variable x_i, which can represent slack ($delL_i$) or strength P_i.

$$x_i = \exp\{ [\ln(-\ln(1-F_w) + \alpha\ln(\beta)] / \alpha\} \tag{D.4}$$
$$0 < F_w < 1$$

The uniform model is a simple two variable distribution with a lower (u_1) and upper (u_2) range. Since the slack is initiated at zero the lower range is zero.

$$F_u(delL;u_1,u_2) = (delL - u_1)/(u_2 - u_1) \tag{D.5}$$
$$0 < F_u < 1$$

solving for $delL_i$ from D.5 and setting $u_1 = 0$

$$delL_i = u_2 F_u \tag{D.6}$$

3. Data Generation and Continuity

A uniform random number generator creates an array of numbers between 0 and 1 and stores it until recalled.

Recalling equation D.3 and substituting in C.1 and solving for the displacement (δ_i).

$$\delta_i = P_i (l + delL_i) / E_f \tag{D.7}$$

The bundle is put under a constant strain rate, so the discrete variable will be displacement. As the bundle is strained the filaments will fail

in order of the underlying strength or strain (neglecting slack and friction). For a constant filament length the strain is proportional to the displacement and the fiber will fail at some underlying displacement. If slack is present in the bundle each fiber will not fail at the underlying displacement because the displacement is no longer proportional to strain but a function of both slack and stain.

The displacement observed by the Instron material testing machine is not δ. The machine records an apparent displacement (d_A)

$$d_{A\,i} = \delta_i + delL_i \tag{D.8}$$

This will affect the fiber failure order, every other variable is then keyed to the order (n_2) of $d_{A\,i}$.

4. Slack Loading

As the bundle is strained, the fibers are loaded by a progressive summation of each fiber in order of their slack distribution. The loading function in the slack region can be defined as a function of the displacement (δ_s) each fiber experiences after slack is released.

$$P_{s\,j} = \sum_{i=1}^{j-1} E_f\, \delta_{s\,i}/L_i \qquad ; j= 1,2,....,n \tag{D.9}$$

$$P_{s\,j} = \sum_{i=1}^{j-1} E_f\, [(delL_{o\,j} - delL_{o\,i})/(1 + delL_{o\,i})] \tag{D.10}$$

where: $delL_{o\,i}$ = order slack

This relationship can be represented graphically in Figure 1D. The last load calculated from equation D.10 will occur at $delL_{o\,n}$.

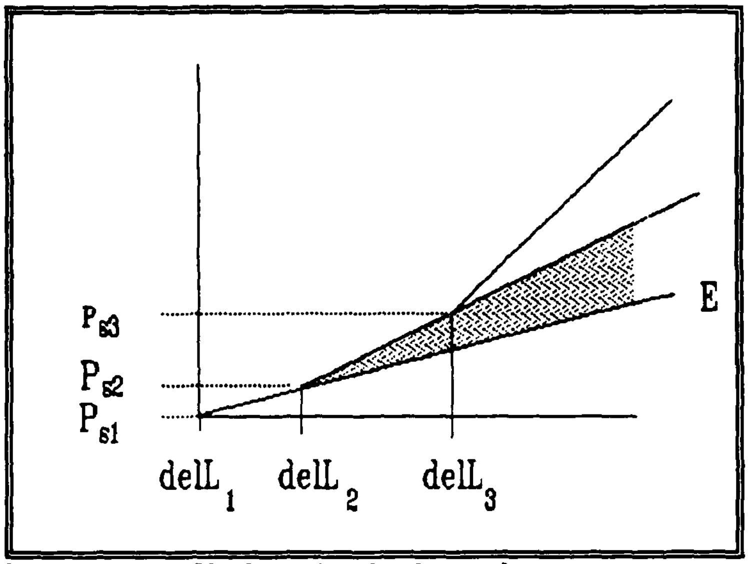

Figure 1D. Slack region load simulation.

4. Failure loading

The order of each fiber failure is determined by the ordered strength distribution (assuming no slack). With slack the order of failure will deviate slightly. The failure load is a function of the modulus of the bundle and the displacement of each fiber. The bundle failure load (P_u) can be defined from equation D.2 as

$$P_{u\,j} = \sum_{i=j}^{n} E_f\ \delta_i/L_i \tag{D.11}$$

substituting in C.1 and D.8

$$P_{u\,j} = \sum_{i=j}^{n} E\ [(d_{A\,j} - delL_i)/(1 + delL_i)] \tag{D.12}$$

5. Discrete Data

The material testing machine measures and transmits data on even intervals of displacement. To simulate this the defined loading points (P_s,$delL_O$) and (P_u,d_A) will be set limits for a linear interpolation. The data is interpolated on intervals of $deld_A$ which is set by the cross head speed. Defining the discrete apparent displacement

$$\underline{d}_{A\,i+1} = \underline{d}_{A\,i} + deld_A \qquad \text{(D.13)}$$

The interpolation algorithm to define the discrete data from continuous is the same for both slack and linear region.

The discrete load $\underline{P}$ in the slack region is a function of $\underline{P}_s$, the ordered slack ($delL_O$) and the apparent discrete displacement

$$\underline{P}_i = [(P_{s\,j+1} - P_{s\,j})/(delL_{O\,j+1} - delL_{O\,j})]\underline{d}_{A\,i} \qquad \text{(D.14)}$$

The discrete data is defined this way until the first underlying displacement $d_{A\,i}$.

The slope of the linear region intercepts the abscissas at a point d0. The slope of the linear region is the bundle modulus $\underline{E}$ (see Figure 2C).

$$\underline{E} = (P_{u\,1} - P_{s\,n})/(d_{A\,1} - delL_n) \qquad \text{(D.15)}$$

The slope can be solve using $\underline{E}$

$$d0 = delL_n - P_{s\,n}/\underline{E} \qquad \text{(D.16)}$$

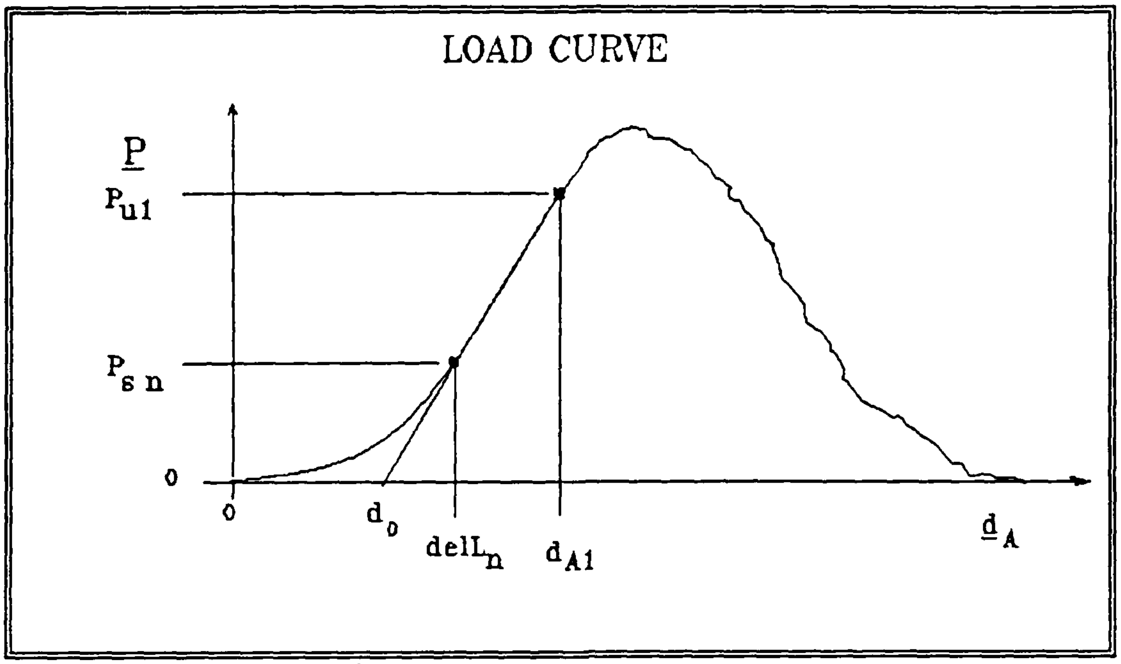

Figure 2C. Linear calculations.

Subsequent discrete data points can be created by a regressive process. At each fiber loss a new slope emanates radial outward from d0. Between breaks, to define the discrete data, the slope is defined by the failure load P_u, the displacement d_A and d0

$$\underline{P}_i = [P_{uj}/(d_{Aj} - d0)]\ (\underline{d}_{Ai} - d0) \qquad \text{(D.17)}$$

where: $\underline{d}_{Ai}$ d_{Aj} ; j=1,2,....,n

6. Noise

Noise was simulated simply by setting a tolerance level and using the random number generator to produce a set of random numbers between -0.5 and 0.5. This data was added to each discrete load $\underline{P}$.

The input file to the simulation has units of kg,mm and the experimental output from the Instron Software is in lbm,in. A conversion of the data is required, this is accomplished through a separate program

B. SIMULATION INTERPRETATION

The simulation was written in fortran code, listed in Appendix F. Multiple simulations were run varying the parameters slack and noise to see the effect.

Simulation data was analyzed via the algorithm as represented in Appendix C. The output is the bundle modulus E, the shape parameter of distribution Alpha, and the location parameter Beta (mm/mm).

The input parameters were alpha = 4.0, beta = 0.16 kg and a 50.0 mm Gage length (GL). The noise was varied from zero to about 0.1% fluctuation of the maximum load. The maximum slack input was 3% of the gage length and a normal slack distribution. The seed used for the random number generator was held constant so the underlying strength distribution was constant. The seed for the slack region and noise was not held constant. The results of multiple simulation test is in Table IC. A value of 4.134 for the shape parameter was retrieved consistently (same seed), as expect no deviation for the same data. The increase in slack drove the parameter down. The noise did not have any effect on the parameters. Figure 3C show the loading curve with slack (minus d0) and no slack.

No noticeable change in the parameter output nor in the ECDF plot (Figure 4C).

Table IC. SIMULATION OBSERVATIONS AND PROOF TESTING RESULTS.

Maximum Slack σ% GL ($\beta = \sigma/2$)	alpha	beta (kg)
0	4.134	0.016
5	4.010	0.016
10	4.099	0.016
	4.095	
	4.091	
	4.089	
	4.088	
	4.075	
	4.076	

The numerical derivative of the slack region is very sensitive to the any noise. This can be seen in the plot of the slack ECDF Figure 5C. In order to retrieve the slack distribution the recording system must have a low tolerance load cell.

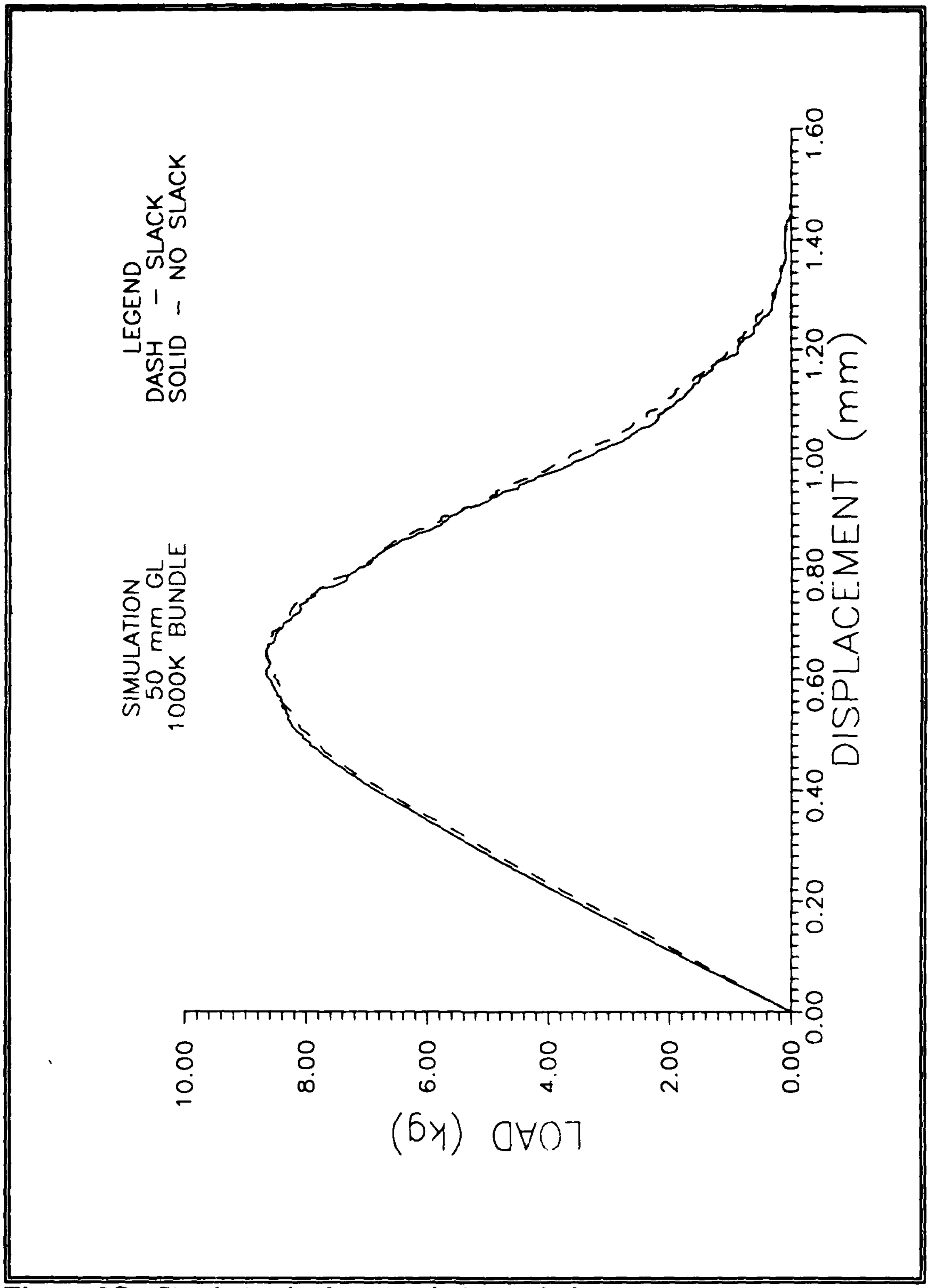

Figure 3C. Simulation load curve, slack; no slack.

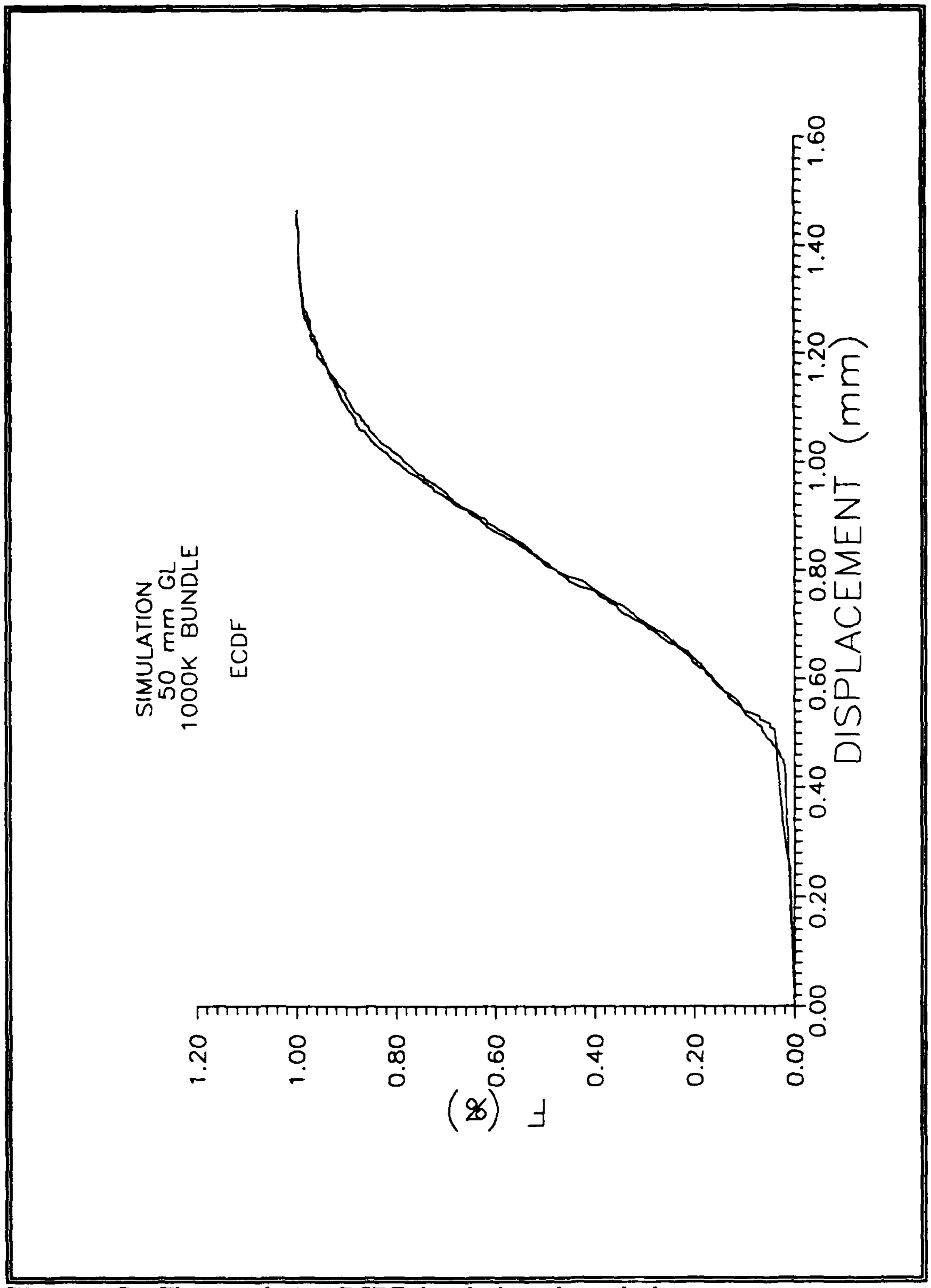

Figure 4C. The simulation ECDF for slack and no slack.

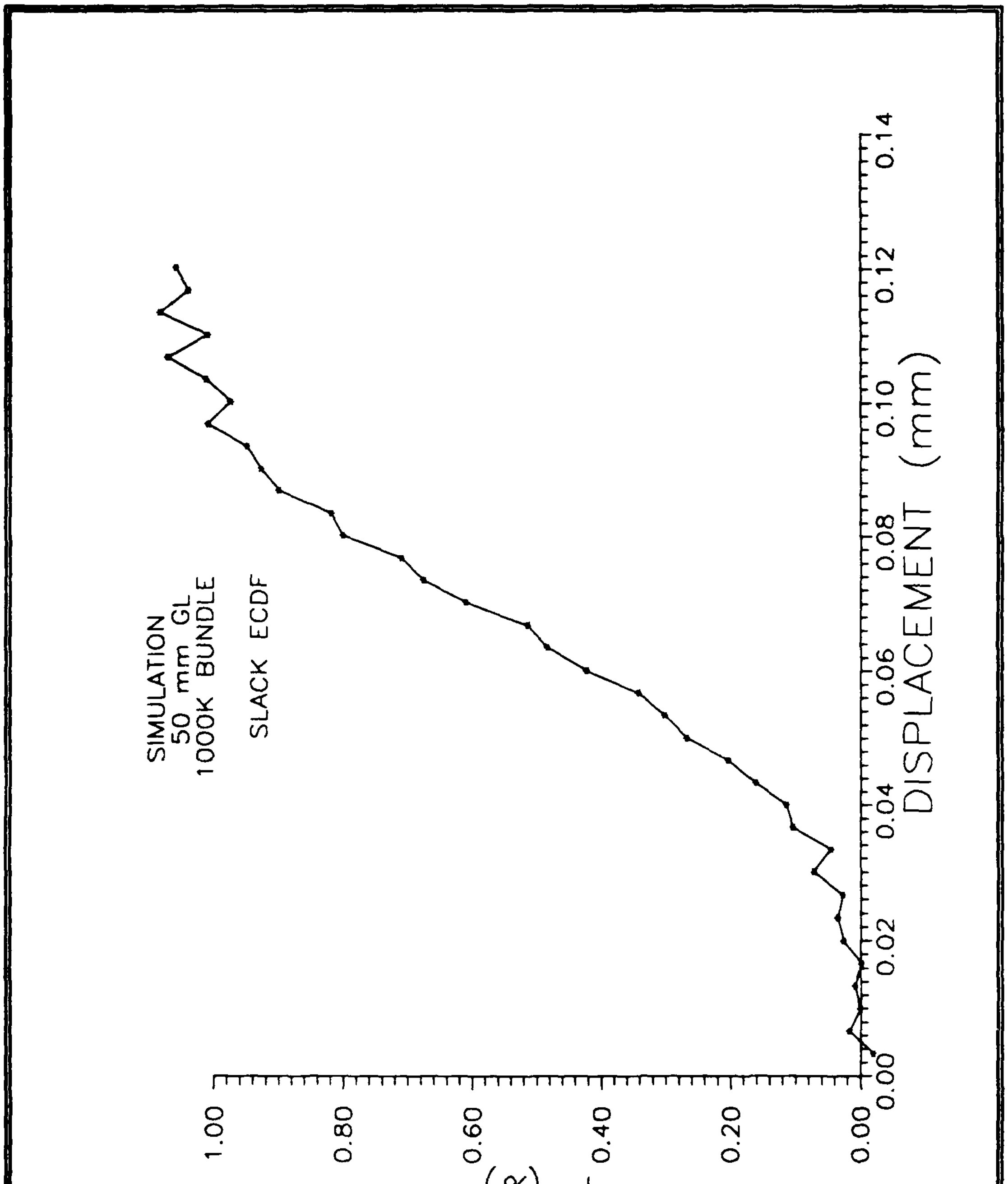

Figure 5C. Simulation of the slack ECDF. Weibull approximation of Normal.

APPENDIX E

I. DATA ANALYSIS CODE

A. ANALYSIS PROGRAM

1. Data Input

The program requires a formatted data file called EXPER.OUT. The data is read in and prompts the user for several variables. These are listed below.

- What is the gage length?
- How many fibers in the bundle?
- Which Numerical differentiation is desired? 3,5,7 pt. forward diff. or 3,5 pt. central diff.
- The end of the slack region and the beginning of the failure region in displacement (mm) (this requires a graphical estimate).
- How many points in the input data file?
- The acquisition tolerance of the displacement? (this is for any filtering of data points in the event the sampling rate is too high and repeated data is evidenced.
- The modulus is calculated and displayed and then ask if this value needs to be change. This is in the event the modulus calculation in correct a manual entry can be made. (Be Careful not to corrupt the true data).

2. Data Output

The data out put is to the screen and to a file. The following is a list of the file outputs.

- EXPER.OUT, the input file is output with an added column of strain
- SLACK.OUT, the slack region (load vs displacement)

- EXMS.OUT, the load curve minus slack region
- UBFECDF.OUT, the Upper bound ECDF
- FECDF.OUT, the ECDF
- SECDF.OUT, the slack region ECDF

B. MAXIMUM LIKELIHOOD ESTIMATOR

The MLE program reads one of the ECDF data files, users choice, the first line in the input file is a utility line defining the file size, the number of filaments and the bundle modulus $\underline{E}$, the remaining lines are data.

1. Data Input

The input other than the read file is prompted from the screen. The following is a list of requested information.

- How many points for the Expected Ranking (The MLE requires a probability ranking)
- First approximation for alpha

2. Data Output

The parameter estimations alpha and beta (mm/mm) are output to the screen.

C. LINEARIZED PLOT

This program takes the ECDF and does a Weibull linearizing (equation C.20).

II. FORTRAN CODE

A. ANALYSIS PROGRAM

```
      PROGRAM ANALY
*           BUNDLE FAILURE ANALYSIS
*
*
* Outline of program:
*        Read in the data points
*        NIA - Number In Array
**********************************************************
*     Find the
*        -linear region
*        -slope of the linear region = EB (E BAR)
*        -intercept point of the linear region with
*         the abscissas = DO  (d zero)
*        -Maximum slack or minimum linear displacement - DP1
*        -Maximum displacement of the linear region - DP2
**********************************************************
*
*  Input the range of the linear region by manually retrieving
*  the defining points DP1 and DP2.
*
**********************************************************
* OUTPUT:    SLACK.OUT    - slack region.
*            SECDF.OUT    - slack ECDF.
*            EXMS.OUT     - experimental data minus slack region.
*            FECDF.OUT    - failure region ECDF.
*            UBFECDF.OUT  - upper bound of the failure region ECDF.
*
      INTEGER NIA,NISA
      PARAMETER(NIA=6000,NISA=1000)
      REAL D(NIA),P(NIA),PP(NIA),H,LP(NIA),LD(NIA),FS(NIA),F(NIA)
     +,FUB(NIA),DF(NIA),DFUB(NIA),PF(NIA),PFUB(NIA),TEMP(NIA),GL
      INTEGER COUNT,NIDP1,NIDP2,NI(NIA),SLCT
      CHARACTER*1, Q1,Q2
*
*************************************************************
* Input analysis control parameters.
*
*************************************************************
         PRINT *, 'What is the gage length? (mm)'
         READ *, GL
*
         PRINT *, ' How many filements?'
         READ *, N
         PRINT *, ' Which method do you require for reduction of'
         PRINT *, ' the slack region to the ecdf space?'
```

```
        PRINT *, '      1) 3 PT forward diff. method.'
        PRINT *, '      2) 5 PT forward diff. method.'
        PRINT *, '      3) 7 PT forward diff. method.'
        PRINT *, '      4) 3 PT central diff. method.'
        PRINT *, '      5) 5 PT central diff. method.'
        READ *, SLCT
*
*****************************************************************
* Read the range of the linear displacement.
*
*****************************************************************
*
         PRINT *, 'INPUT DP1 (the maximum slack)'
         READ *, DP1
         PRINT *, 'INPUT DP2 (the maximum linear displacement)'
         READ *, DP2
********************************************************************
* Find the how many points are in the data file.
*
********************************************************************
         PRINT*, 'How many points in the data file?'
         READ *, COUNT
********************************************************************
*  Find out what the tolerance of the machine is.
*  (displacement)
********************************************************************
         PRINT *, 'What is the tolerance of the displacement output'
         PRINT *, 'data? This is the limit between data displacement'
         PRINT *, 'Input in mm.'
         READ *, TOLD
         PRINT *, 'How many points in the empirical rank?'
         READ *, NER
144       PRINT *, 'Do you want the load in Kg (K) or Newtons (N)?'
         READ(*,303) Q2
         IF(Q2 .EQ. 'K' .OR. Q2 .EQ. 'N') THEN
          PRINT *, ' '
         ELSE
          PRINT *, 'Try again in UPPER CASE K OR N'
         GOTO 144
         ENDIF
*
********************************************************************
* Read in the experimental results.
*
********************************************************************
*
      OPEN(UNIT=11,FILE='EXPER.OUT',STATUS='OLD')
*
      DO 10 I=1,COUNT
      READ(11,110) NI(I),TEMP(I),P(I)
10     CONTINUE
      REWIND(11)
```

```
      CONVER = 1.0
      IF(Q2 .EQ. 'N') THEN
      CONVER = 9.8062
      ENDIF
      DO 44 I=1,COUNT
      P(I) = P(I)*CONVER
      WRITE(11,1001) NI(I),TEMP(I),P(I),TEMP(I)/GL
44    CONTINUE
*******************************************************************
* Filter out any points with difference in displacement interval
* less than the machine tolerance TOLD.
*
*******************************************************************
      J=1
      D(1) = TEMP(1)
      DO 11 I=2,COUNT
      DIFF = TEMP(I)-TEMP(I-1)
      IF(DIFF .LT. TOLD) THEN
       PRINT *, 'SKIPPED POINT ',I
      ELSE
       J=J+1
       D(J) = TEMP(I)
       P(J) = P(I)
      ENDIF
11    CONTINUE
       COUNT = J
*
*      *      *      *      *      *      *      *
*   FIND COUNT (NI) OF DP1 AND DP2
*      *      *      *      *      *      *      *
*
          I=1
20        IF(D(I) .LT. DP1) THEN
           I=I+1
           GOTO 20
          ELSE
           NIDP1 = I
           DP1 = D(I)
          ENDIF
30        IF(D(I) .LT. DP2) THEN
           I=I+1
           GOTO 30
          ELSE
           NIDP2 = I-1
           DP2 = D(I-1)
          ENDIF
*
*******************************************************************
* OUTPUT THE SLACK REGION.
*
*******************************************************************
*
```

```
      OPEN(UNIT=13,FILE='SLACK.OUT',STATUS='UNKNOWN')
      DO 500 I = 1,NIDP1
      WRITE(13,110) I,D(I),P(I)
500   CONTINUE
      CLOSE(13)
*
*******************************************************************
*  LEAST SQUARES TO DETERMINE THE SLOPE BETWEEN DP1 AND
DP2.
*
*******************************************************************
*
      TOP = 0.0
      BOT = 0.0
      DO 40 I=(NIDP1+1),NIDP2
         LP(I) = P(I) - P(NIDP1)
         LD(I) = D(I) - DP1
         TOP = TOP + LP(I)*LD(I)
         BOT = BOT + LD(I)**2
40    CONTINUE
      EB = TOP/BOT
*     *     *     *     *     *     *     **
* Find the intercept point do
*     *     *     *     *     *     *     **
*
      D0 = DP1 - P(NIDP1)/EB
      PRINT *, 'Do = ',D0
      PRINT *, ' '
      PRINT *, 'The Modulus calculated from the load curve'
      PRINT *, 'is ',EB
      PRINT *, ' '
      PRINT *, 'Would you like to change this (Y/N)?'
      READ(*,303) Q1
      IF(Q1 .EQ. 'Y' .OR. Q1 .EQ. 'y') THEN
      PRINT *, 'Input the new modulus'
      READ *, EB
      ENDIF
303   FORMAT(A1)
*
*
*******************************************************************
* OUTPUT THE EXPERIMENTAL DATA MINUS SLACK.
*
*******************************************************************
      OPEN(UNIT=13,FILE='EXMS.OUT',STATUS='UNKNOWN')
      J=1
      WRITE(13,110) J,0.0,0.0
      DO 550 I = NIDP1,COUNT
      J=J+1
      WRITE(13,110) J,(D(I)-D0),P(I)
550   CONTINUE
*
```

```
*****************************************************************
* NUMERICAL DIFFERENTIATION OF THE SLACK REGION
*     (using a forward and central difference methods).
*
*****************************************************************
*
      IF(SLCT .EQ. 1) GOTO 103
      IF(SLCT .EQ. 2) GOTO 105
      IF(SLCT .EQ. 3) GOTO 107
      IF(SLCT .EQ. 4) GOTO 130
      IF(SLCT .EQ. 5) GOTO 150
*
*********************************
*  3 pt forward diff. method.
*********************************
*
103      DO 3 I=1,NIDP1
* if out of the slack region leave the loop
         IF (D(I) .GT. DP1) GOTO 51
      H = ABS(D(I+1)-D(I))
* 3 point forward difference numerical differentiation.
* Generate Fs(da) CDF
      PP(I) = (-3*P(I)+4*P(I+1)-P(I+2))/(2*H)
      FS(I) = PP(I)/EB
3     CONTINUE
      GOTO 51
*********************************
* 5 pt forward difference method.
*********************************
105      DO 5 I=1,NIDP1
* if out of the slack region leave the loop
         IF (D(I) .GT. DP1) GOTO 51
      H = ABS(D(I+1)-D(I))
* 5 point forward difference numerical differentiation.
* Generate Fs(da) CDF
      PP(I) = (-25*P(I)+48*P(I+1)-36*P(I+2)+16*P(I+3)-
     C3*P(I+4))/(12*H)
      FS(I) = PP(I)/EB
5     CONTINUE
*********************************
* 7 pt forward difference method
*********************************
107      DO 7 I=1,NIDP1
* if out of the slack region leave the loop
         IF (D(I) .GT. DP1) GOTO 51
       H = ABS(D(I+1)-D(I))
* 7 point forward difference numerical differentiation.
* Generate Fs(da) CDF
      PP(I) = (-147*P(I)+360*P(I+1)-450*P(I+2)+
     +400*P(I+3)-225*P(I+4)+72*P(I+5)-10*P(I+6))/(60*H)
      FS(I) = PP(I)/EB
7     CONTINUE
```

```
**********************************
* 3 pt central difference method
**********************************
*
130      I=1
         H = ABS(D(I+1)-D(I))
      PP(I) = (-3*P(I)+4*P(I+1)-P(I+3))/(2*H)
      FS(I) = PP(I)/EB
      DO 33 I=2,NIDP1
* if out of the slack region leave the loop
        IF (D(I) .GT. DP1) GOTO 51
      H = ABS(D(I+1)-D(I))
      PP(I) = (-P(I-1)+P(I+1))/(2*H)
* Generate Fs(da) CDF
      FS(I) = PP(I)/EB
33    CONTINUE
**********************************
* 5 pt central difference method
**********************************
*
150      I=1
*  Use a 3 pt forward difference method for first couple pts.
         H = ABS(D(I+1)-D(I))
         PP(I) = (-3*P(I)+4*P(I+1)-P(I+3))/(2*H)
         FS(I) = PP(I)/EB
         I=2
         H = ABS(D(I+1)-D(I))
         PP(I) = (-3*P(I)+4*P(I+1)-P(I+3))/(2*H)
         FS(I) = PP(I)/EB
       DO 55 I=3,NIDP1
* if out of the slack region leave the loop
       IF (D(I) .GT. DP1) GOTO 51
         H = ABS(D(I+1)-D(I))
      PP(I) = (P(I-2)-8*P(I-1)+8*P(I+1)-P(I+2))/(12*H)
* Generate Fs(da) CDF
      FS(I) = PP(I)/EB
55    CONTINUE
**********************************
51    CONTINUE
*****************************************************************
*  FAILURE REGION
*
*****************************************************************
      J=0
      DO 60 I=NIDP2,COUNT
      J=J+1
      F(J) = 1 - P(I)/(EB*(D(I)-D0))
      DF(J) = D(I)-D0
60    CONTINUE
      NUMF = J
*****************************************************************
* EXTRACT THE UPPER BOUND.
```

```
*
*****************************************************************
      DEL = 1.0/NER
      K=NUMF
      I=NER
         FUB(I+1) = F(K)
         DFUB(I+1) = DF(K)
         K=K-1
         I=I-1
      FL =  1.0*I/NER - DEL/2
      FU =  1.0*I/NER + DEL/2
      PRINT *,'FL        F(K)        FU'
      PRINT *, FL,' < ',F(K),' < ',FU
90     IF((K .LT. 1) .OR. (I .LT. 0)) GOTO 92
91     IF((F(K) .LT. FU) .AND. (F(K) .GE. FL)) THEN
         FUB(I+1) = F(K)
         DFUB(I+1) = DF(K)
           I=I-1
         FL = 1.0*I/NER - DEL/2
         FU = 1.0*I/NER + DEL/2
           K=K-1
           IF(K .LT. 1) GOTO 92
           GOTO 91
       ELSEIF(F(K) .LT. FL) THEN
*      *      *      *      *      *      *      *
*         check to see how many it skips
*      *      *      *      *      *      *      *
          J1 = 1
 93       IF(F(K) .LT. (1.0*(I-J1)/NER - DEL/2)) THEN
           J1=J1+1
           GOTO 93
           ENDIF
           DO 94 I1=I,(I-J1+1),-1
           IF(I1+1 .LT. 1) GOTO 92
           FUB(I1+1) = 1.0*I1/NER
           DFUB(I1+1) = DF(K)
94        CONTINUE
           I=I1
           FUB(I+1)=F(K)
           DFUB(I+1)=DF(K)
           I=I-1
         FL = 1.0*I/NER - DEL/2
         FU = 1.0*I/NER + DEL/2
           K=K-1
           GOTO 90
        ELSE
         K=K-1
         IF(K .LT. 1) GOTO 92
         GOTO 90
        ENDIF
92     CONTINUE
       PRINT *, 'OUT OF LOOP WITH K AND I '
```

```
      PRINT *, 'K= ',K,'  I= ',I
      NUMEF = I+2
*************************************************************
* WRITE THE DATA
*
*************************************************************
      OPEN (UNIT=11,FILE='UBFECDF.OUT',STATUS='UNKNOWN')
      MSUM = NER+1-NUMEF
      WRITE(11,1010) MSUM,N,EB
*
      DO 250 I=NUMEF,NER
      WRITE(11,110) I,DFUB(I),FUB(I)
250     CONTINUE
      CLOSE(11)
     OPEN(UNIT=11,FILE='FECDF.OUT',STATUS='UNKNOWN')
      WRITE(11,1010) NUMF,N,EB
     DO 70 I=1,NUMF
     WRITE(11,110) I,DF(I),F(I)
70    CONTINUE
     CLOSE(11)
    OPEN(UNIT=12,FILE='SECDF.OUT',STATUS='UNKNOWN')
    DO 80 I=1,NIDP1
    WRITE(12,110) I,D(I),FS(I)
80    CONTINUE
*******************************************************************
* FORMATS
*
*******************************************************************
100   FORMAT(1X,I5)
110   FORMAT(1X,I5,2X,F8.4,2X,F8.4)
1001  FORMAT(1X,I5,2X,F8.4,2X,F8.4,2X,E10.4)
1000  FORMAT(1X,I5,2X,E16.10,2X,E16.10)
1010  FORMAT(1X,I5,2X,I5,2X,F8.4)
***********************************************************
    END
```

B. MAXIMUM LIKELIHOOD ESTIMATOR

```
      PROGRAM MLE
      INTEGER NIA
      PARAMETER (NIA = 6000)
      REAL FE(NIA),DFE(NIA),FUB(NIA),DFUB(NIA),GL
      CHARACTER*1 Q1
*
      PRINT *, 'Do you want the upper bound or complete data'
      PRINT *, 'set, suggest the compete data set when the'
      PRINT *, 'DATA is less the the number of filements'
110    PRINT *, 'TYPE "U" for upper and "C" otherwise'
303    FORMAT(A1)
      READ (*,303) Q1
      IF(Q1 .EQ. 'U') THEN
      OPEN(UNIT=11,FILE='UBFECDF.OUT',STATUS='OLD')
```

```
      ELSEIF (Q1 .EQ. 'C') THEN
      OPEN(UNIT=11,FILE='FECDF.OUT',STATUS='OLD')
      ELSE
      PRINT *, 'TRY TYPING CAPITAL LETTERS'
      GOTO 110
      ENDIF
*
      READ(11,1010) M,N,EB
1010   FORMAT(1X,I5,2X,I5,2X,F8.4)
      DO 10 I=1,M
      READ(11,1020) J,DFUB(I),FUB(I)
10     CONTINUE
1020   FORMAT(1X,I5,2X,F8.4,2X,F8.4)
************************************************************
* CALCULATE ALPHA AND BETA
      PRINT *, 'What is the GAGE LENGTH?'
      READ *, GL
*
* REDUCE THE DATA TO ABOUT 50 POINTS
888    PRINT *, 'How many points for the MLE ?'
      READ *, MLE
      PRINT *, MLE
      DEL = 1.0/(MLE+1.0)
      J = 1
      I =1
* find the value of F above and below the expected rank and average
400    FE(I) = 1.0*I/(MLE+1.0)
      IF(I .GT. MLE) GOTO 450
410    IF((FUB(J) .LE. FE(I)) .AND. (FUB(J+1) .GT. FE(I))) THEN
      DFE(I)= -(DFUB(J+1)-DFUB(J))*(FUB(J+1)-FE(I))/(FUB(J+1)-FUB(J))
     +  + DFUB(J+1)
      I=I+1
      GOTO 400
      ELSEIF((J+1) .LE. (M)) THEN
      J=J+1
      GOTO 410
      ENDIF
450    PRINT *, I,J,FUB(J),FUB(J+1),FE(I)
      IF(FUB(J+1) .LE. 0.0) THEN
       PRINT *, 'MLE to large try again'
       GOTO 888
      ENDIF
************************************************************
* estimate the initial alpha
      PRINT *, 'Estimate the intial alpha'
      READ *,   ALFA0
************************************************************
* calculate
       ALFA = ALFA0
       KK=0
320     DO 310 IT = 1,2
       SUM1=0.0
```

```
      SUM2=0.0
      SUM3=0.0
* Calculate the variables of alpha
      DO 300 I=1,MLE
       IF(DFE(I) .GT. 0.0) THEN
      SUM1 = SUM1 + ((DFE(I))**ALFA * LOG(DFE(I)))
      SUM2 = SUM2 + ((DFE(I))**ALFA)
      SUM3 = SUM3 + LOG(DFE(I))
       ENDIF
300    CONTINUE
      IF(IT .EQ. 1) THEN
       ALFA1=1.0/(SUM1/SUM2 - (SUM3/MLE))
       ALFA = ALFA1
      ELSEIF(IT .EQ. 2) THEN
       ALFA2=1.0/(SUM1/SUM2 - (SUM3/MLE))
      ENDIF
310    CONTINUE
      ALFA0P = ALFA0 -((ALFA0 - ALFA1)**2)/(ALFA0 - 2*ALFA1
    + + ALFA2)
      TEST = ABS(ALFA0P-ALFA0)
      IF(TEST .GT. .0001) THEN
       IF(KK .GT. 1000) GOTO 341
      KK = KK + 1
      ALFA0 = ALFA0P
     PRINT *, 'ALFA0 =',ALFA0
      GOTO 320
      ELSE
      ALPHA = ALFA0P
      ENDIF
      SUM = 0.0
      DO 340 I = 1,MLE
      SUM = SUM + (DFE(I))**ALPHA
340    CONTINUE
341       BETA = (SUM/(MLE))**(1/ALPHA)
      PRINT *, 'ALPHA = ',ALPHA,'   BETA = ',BETA/GL,' mm/mm'
      END
```

C. WEIBULL LINEAR CDF

```
      PROGRAM LIN
      INTEGER NIA,AREA
      PARAMETER (NIA = 6000)
      REAL FE(NIA),DFE(NIA),FUB(NIA),DFUB(NIA),FS(NIA),D(NIA)
*
      OPEN(UNIT=11,FILE='UBFECDF.OUT',STATUS='OLD')
      READ(11,1010) M,N,EB
1010   FORMAT(1X,I5,2X,I5,2X,F8.4)
      DO 10 I=1,M
      READ(11,1020) J,DFUB(I),FUB(I)
10    CONTINUE
1020   FORMAT(1X,I5,2X,F8.4,2X,F8.4)
*************************************************************
```

```
* REDUCE THE DATA TO ABOUT 50 POINTS, USING THE EXPECTED
RANK.
       PRINT *, 'HOW MANY POINTS TO FILTER USING THE EXPECTED
RANK'
       READ *, NUM
       PRINT *, NUM
       DEL = 1.0/(NUM+1.0)
       J = 1
       I =1
* find the value of F above and below the expected rank and average
400     FE(I) = 1.0*I/(NUM+1.0)
       IF(I .GT. NUM) GOTO 450
410     IF((FUB(J) .LE. FE(I)) .AND. (FUB(J+1) .GT. FE(I))) THEN
       DFE(I)= -(DFUB(J+1)-DFUB(J))*(FUB(J+1)-FE(I))/(FUB(J+1)-FUB(J))
     +  + DFUB(J+1)
       I=I+1
       GOTO 400
       ELSEIF((J+1) .LT. (M+1)) THEN
       J=J+1
       GOTO 410
       ENDIF
450     PRINT *, I,J
*****************************************************************
* CALCULATE ALPHA AND BETA
*
* FIND F*=LOG(-LOG(1-F)) VS LOG(DELTA/BETA)
        DO 20 I=1,M
        IF(FUB(I) .LE. 0.0) FUB(I) = 0.0001
        FS(I) = LOG(-LOG(1-FUB(I)))
        IF(DFUB(I) .LE. 0.0) DFUB(I) = 0.0001
        D(I) = LOG(DFUB(I))
20       CONTINUE
      DO 70 I=1,M
510    IF((FS(I) .LE. 0.63212) .AND. (FS(I+1) .GT. 0.63212)) THEN
       BETA = -(D(I+1)-D(I))*(FS(I+1)-0.63212)/(FS(I+1)-FS(I))
     +  + D(I+1)
       GOTO 71
       ENDIF
70       CONTINUE
71       CONTINUE
        BETA = EXP(BETA)
        PRINT *, ' BETA = ',BETA
*
*
        DO 80 I=1,M
        IF(DFUB(I) .LE. 0.0) DFUB(I) = 0.0001
         D(I) = LOG(DFUB(I)/BETA)
80       CONTINUE
         BETA = EB*BETA/N
*****************************************************************
      OPEN(UNIT=11,FILE='FSTAR.OUT',STATUS='UNKNOWN')
      DO 60 I=1,M
```

```
      WRITE(11,1020) I,D(I),FS(I)
60    CONTINUE
      END
```

APPENDIX F

SIMULATION FORTRAN CODE

```
      PROGRAM SIMUL
* ------------------------------------------------------
*     SIMUL IS A STATISTICAL SIMULATION ROUTINE USED
*     FOR RELIABILITY
*     ANALYSIS OF COMPOSITE FILEMENTS.
* ------------------------------------------------------
*    UTILITIES FILES REQUIRED ARE:
*       SEED - file containing the integer seed for random
*              number generation
*       INPUT - file containing the input parameters *
*
*
*    OUTPUT FILES INCLUDE:
*       EXPER.OUT - the experimental simulation of discrete
*                   data.
*       SEED - outputs a new seed.
*
*------------------------------------------------------------
* INPUT FILE FOR BUNDLE2.FOR PROGRAM      :line 1
* XXXXXXXXXXXXXXXXXXXX12345678            :line 2
* E  (kg)             .90                 :line 3
* ROWS =  M           100                 :line 4
* COLS =  N           100                 :line 5
* MGL   (mm)          250.0               :line 6
* ALPHA               4.0                 :line 7
* BETA                16.0                :line 8
* MODEL               NORMAL              :line 9
* SKEW/ALPHA          3.5                 :line 10
* MAX SLACK (%  )     10.0                :line 11  % OF MAX
*                                           DISPLACEMENT
* SAMPLE RATE         20.0                :line 12  pts/s
* CROSS-HEAD SPEED    1.8                 :line 13  mm/min
* NOISE  +/- kg       .001                :line 14  tolerance
* TOLD (mm)           .00254              :line 15  tolerance of displ.
*                                           output
*------------------------------------------------------------
*   INPUT PARAMETERS ARE:
*       1) THE LENGTH OF THE TEST BUNDLE (MGL).
*       2) THE NUMBER OF FILEMENTS PER BUNDLE (N).
*       3) THE NUMBER OF SEGMENTS PER FILEMENT (M).
*       4) THE SEGMENT LENGTH IS DEFINED BY MGL=M*LS
*
*   THIS PROGRAM CREATES:
*
```

```
*       1) THE RANDOM NUMBERS
*       2) CONVERTS TO WIEBULL BUNDLE
*       3) FIND THE WEAKEST IN THE ENTIRE BUNDLE
*          FOR THE LOWER BOUND.
*       4) FIND THE WEAKEST IN EACH FILEMENT
*       5) ORDER THE WEAKEST TO STRONGEST FILEMENTS
*       6) CHANGE THE STRENGTHS TO BREAKING LOADS AND
*          DISPLACEMENTS
************************************************************
* Begin Program:
*
************************************************************
*
      INTEGER ROWS,COLS,MAX,LIM,LIM1,N,PTS,COUNT,LIM2
      PARAMETER(LIM=6000,LIM1=1000,LIM2=1001)
      REAL BUN(LIM1),RAND1(LIM1),RAND2(LIM1),DISPL(LIM1),
     +SLACK(LIM1),P(LIM1),ALPHA,BETA,MGL,BRKLDM,
     +DISPLM,MAXX,MAXY,MEAN,PU(LIM1),
     +PL(LIM1),SUM,MAXSLK,TEMP(LIM2),PS(LIM2),D(LIM),
     +DISPLA(LIM1),DEL,PMS,LOAD(LIM),NSE
      DIMENSION NO(LIM1),NB(LIM1),NDB(LIM1)
      DOUBLE PRECISION SEED
      CHARACTER*1 MODEL*7,SONS,Q,Q2,Q3,Q4
      COMMON MAX
      COMMON /SML/M,N
*
*****************************************************************
*
*  FILE 11 = PARAM DATA FOR INPUT
*  FILE 12 = SEED DATA FOR SEEDING
*
*****************************************************************
*
OPEN(UNIT=11,FILE='INPUT',STATUS='OLD')
OPEN(UNIT=12,FILE='SEED',STATUS='OLD')
*
************************************************************
*     READ IN THE INPUT DATA
*
************************************************************
*
      READ(11,1530)
      READ(11,1500) E
      PRINT *, 'MODULUS=', E
      READ(11,1520) M
      PRINT *, 'M = ',M
      ROWS=M
      READ(11,1520) N
      COLS=N
      PRINT *, 'N = ',N
      READ(11,1510) MGL
      PRINT*, 'Mean Gage Length = ',MGL
```

```
    MAX=M*N
      READ(11,1510) ALPHA
      PRINT *, 'ALPHA = ',ALPHA
     READ(11,1510) BETA
      PRINT *, 'BETA =',BETA
*
*     *     *     *     *     *
* Determine which model to use for slack.
*     *     *     *     *     *
*
      READ(11,1515) MODEL
      PRINT *, 'MODEL=',MODEL,' APPROXIMATION'
       IF (MODEL .EQ. 'NORMAL') THEN
       READ(11,1500) STDDEV
         PRINT *, 'ALPHA = ',STDDEV
         READ(11,1510) MEAN
       PRINT *, 'Maximum slack as percent of expected'
       PRINT *, 'maximum displacement (3% gage length).=',MEAN
       MEAN = (MEAN/100)*(0.03)*MGL/2
       PRINT *, 'Mean=',MEAN,'mm'
       PRINT *, 'Maximum slack =',MEAN*2,'mm'
       ELSE
         READ(11,1500) RANGE1
         READ(11,1500) RANGE2
       RANGE1=(RANGE1/100)*(.03)*MGL
       RANGE2=(RANGE2/100)*(.03)*MGL
         PRINT *, 'RANGE 1 =',RANGE1,' mm'
         PRINT *, 'RANGE 2 =',RANGE2,' mm'
       ENDIF
*
*     *     *     *     *     *
* Read in the sampling rate and cross-head speed.
*     *     *     *     *     *
*****************************************************************
* Ensure the displacement interval governed by the cross-head and
* sampling rate is greater than the tolerance level of the output
* displacement data.
*
*****************************************************************
*
      READ(11,1510) SR
       PRINT *, 'Sampling rate =',SR
      READ(11,1510) XSP
       PRINT *, 'Cross-head speed =',XSP
*
*     *     *     *     *     *
* How much noise.
*     *     *     *     *     *
*
      READ(11,1510) NSE
       PRINT *, 'Range of noise =',NSE
      READ(11,1510) TOLD
```

```
      PRINT *, 'Tolerance of output =',TOLD
**       *        *        *        *        *        *        *
*  Calculate the number of total points recorded in the      *
*  experiment.                                                 *
**       *        *        *        *        *        *        *
*
444   RR = MGL*(0.03)
      PTS = RR*SR/(XSP/60)
      DELTAD = RR/PTS
      IF(TOLD .GT. DELTAD) THEN
       PRINT *, 'The displacement interval is less than the output'
       PRINT *, 'tolerance, please input a new cross-head and/or'
       PRINT *, 'sampling rate.'
       PRINT *, 'Current Cross-head = ',XSP
       PRINT *, 'Input new XSP'
       READ *, XSP
       PRINT *, 'Current sampling rate = ',SR
       PRINT *, 'Input new sampling rate'
       READ *, SR
       GOTO 444
       ENDIF
*
*      *        *        *        *        *
*  Determine the type of simulation.
*      *        *        *        *        *
*
       PRINT *, 'SLACK OR NO SLACK ? (Y/N)'
       READ (*,1600) SONS
       PRINT *, 'NOISE OR NO NOISE ? (Y/N)'
       READ (*,1600) Q2
       PRINT *, 'Change the SEED ?'
       READ (*,1600) Q
       PRINT *, 'COMPLAINCE (Y/N)?'
       READ (*,1600) Q3
       IF((Q3 .EQ. 'Y') .OR. (Q3 .EQ. 'y')) THEN
       PRINT *, 'WHAT IS THE COMPLIANCE NOISE LEVEL?'
       READ *, CTOL
       ENDIF
      PRINT *, 'Do you want English units (lbs,inches) on output?
     +(Y/N)'
      PRINT *, 'If N the output is in SI units (kg,mm)'
       READ (*,1600) Q4
*
      CLOSE(11)
*
```

```
**********************************************************************
*  BEGIN THE ALGORITHM                                                  **
**********************************************************************
**
**      *       *       *       *       *       *       *
* Read the seed.  (seed for the uniform random number generator).
* and generate the random numbers
**      *       *       *       *       *       *       *
*
      READ(12,1000) SEED
      REWIND(12)
      DO 10 I=1,N
        CALL RAND(SEED,M,BUN)
        CALL SMALL(ROWS,BUN,RAND1(I))
10    CONTINUE
*
**      *       *       *       *       *       *       *
* Check if slack is requested in simulation.
**      *       *       *       *       *       *       *
*
        IF (SONS .EQ. 'Y') THEN
         CALL RAND(SEED,N,RAND2)
        ENDIF
*
**      *       *       *       *       *       *       *
* Initialize the order arrays.
**      *       *       *       *       *       *       *
*
        DO 25 I=1,N
         SLACK(I)=0.0
         NO(I)=I
         NB(I)=I
         NDB(I)=I
25      CONTINUE
*
**      *       *       *       *       *       *       *
* Check if seed is to be changed, if so write the new seed.
**      *       *       *       *       *       *       *

*
        IF ((Q .EQ. 'Y') .OR. (Q .EQ. 'y')) THEN
        WRITE(12,1000) SEED
        REWIND (12)
        CLOSE(12)
        ENDIF
*
**********************************************************************
* Generate the load and slack distributions.
*
**********************************************************************
*
```

```
        CALL WEIBUL(ALPHA,BETA,N,RAND1,P)

       IF (SONS .EQ. 'Y' .OR. SONS .EQ. 'y') THEN
        IF (MODEL .EQ. 'NORMAL') THEN
         CALL NORMAL(STDDEV,MEAN,N,RAND2,SLACK,MAXSLK)
        ELSE
         CALL UNIFM(RANGE1,RANGE2,N,RAND2,SLACK,MAXSLK)
        ENDIF
        SLACK(1)=0.0
      ENDIF
*
*****************************************************************
* Find the displacement for each failure load and slack.
*
*****************************************************************
*
       DO 20 I=1,N
       DISPL(I) = (P(I)/E)*(SLACK(I) + MGL)
20      CONTINUE
        K=N
*     *     *     *     *     *
*  Order the "DISPL" array, and carry "No"
*     *     *     *     *     *
*
       CALL SHELL(N,DISPL,NO)
*
*     *     *     *     *     *
* reorder the remaining arrays, the others are
* keyed to "DISPL"
*     *     *     *     *     *
*
      CALL SWITCH(N,NO,RAND1)
      CALL SWITCH(N,NO,RAND2)
      CALL SWITCH(N,NO,SLACK)
      CALL SWITCH(N,NO,P)
*
*****************************************************************
*  Calculate the apparent displacement.
*****************************************************************
*
        DO 40 I=1,N
        DISPLA(I) = DISPL(I) + SLACK(I)
40       CONTINUE
*
**     *     *     *     *     *
* Reorder the arrays keyed to "DISPLA"
**     *     *     *     *     *
*
      CALL SHELL(N,DISPLA,NB)
      CALL SWITCH(N,NB,RAND1)
      CALL SWITCH(N,NB,RAND2)
      CALL SWITCH(N,NB,SLACK)
```

```
      CALL SWITCH(N,NB,DISPL)
      CALL SWITCH(N,NB,P)
      CALL SWCH2(N,NB,NO)
*
**      *      *      *      *      *
* Define the ordered slack as "TEMP"
**      *      *      *      *      *
*
      DO 45 I=1,N
      TEMP(I)=SLACK(I)
45    CONTINUE
      IF((SONS .EQ. 'Y') .OR. (SONS .EQ. 'y')) THEN
      CALL SHELL(N,TEMP,NDB)
      ENDIF
*
*********************************************************************
*                                                                  *
*  Calculate the maximum slack and intercept point.                 *
*  Calculate the upper and lower Pi bundle load at each displa.     *
*                                                                  *
*                                                                  *
*********************************************************************
* Find the loads at the slack points.
**      *      *      *      *      *
*
      PS(1)=0.0
      DO 70 J=2,N
      CONST=0.0
       DO 75 I=1,(J-1)
        CONST=CONST+E*(TEMP(J)-TEMP(I))/(MGL+TEMP(I))
75     CONTINUE
      PS(J)=CONST
70    CONTINUE
*********************************************************************
* ADD COMPLIANCE TO SLACK                                              *
*                                                                  *
* DC1 = mm                                                            *
*********************************************************************
      IF((Q3 .EQ. 'Y') .OR. (Q3 .EQ. 'y')) THEN
      DC1 = .0356
      A1=  0.0512
      A2= -0.0217
      POWER = 0.1975
      A3 = 0.0064
      A4 = 0.0242
      DO 120 I = 1,N
      IF((TEMP(I) .LT. DC1) .AND. (PS(I) .GE. CTOL)) THEN
      TEMP(I) = TEMP(I) + A1*(PS(I)**POWER) + A2
       ELSEIF(PS(I) .GE. .01) THEN
      TEMP(I) = TEMP(I) + A3*PS(I) + A4
       ENDIF
120   CONTINUE
```

```
      ENDIF
*
********************************************************************
* FAILURE POINTS.
********************************************************************
*
       DO 80 J=1,N
          CONST=0.0
       DO 85 I=J,N
        CONST=CONST+E*(DISPLA(J)-SLACK(I))/(MGL+SLACK(I))
85          CONTINUE
          PU(J)=CONST
80        CONTINUE
********************************************************************
* ADD COMPLIANCE TO FAILURE REGION                                      *
*                                                              *
* DC1 = mm                                                         *
********************************************************************
      IF((Q3 .EQ. 'Y') .OR. (Q3 .EQ. 'y')) THEN
      DO 121 I = 1,N
      IF((DISPLA(I) .LT. DC1) .AND. (PU(I) .GE. CTOL)) THEN
      DISPLA(I) = DISPLA(I) + A1*(PU(I)**POWER) + A2
       ELSEIF(PU(I) .GE. .01) THEN
      DISPLA(I) = DISPLA(I) + A3*PU(I) + A4
       ENDIF
121    CONTINUE
      ENDIF
*
*******************************************************************
* ADD DISCRETE DATA                                               *
* TO THE                                                     *
* SLACK REGION.                                                *
*                                                           *
*******************************************************************
*
      J=1
      ADD = 0.0
      CONST=0.0
      D(J)=DELTAD
      DO 90 I=1,(N-1)
       IF (D(J) .LT. TEMP(I+1)) THEN
       SLOPE = (PS(I+1)-PS(I))/(TEMP(I+1)-TEMP(I))
91      LOAD(J)=(D(J)-TEMP(I))*SLOPE + ADD
       CONST=D(J)
       J = J + 1
       D(J)=CONST+DELTAD
         IF (D(J) .LT. TEMP(I+1)) GOTO 91
         ENDIF
       ADD = ADD + PS(I+1)-PS(I)
90     CONTINUE
      JBIGL = J-1
*
```

```
*******************************************************************
* ADD DISCRETE DATA                                               *
* TO THE                                                      *
* LINEAR REGION                                                 *
*                                                            *
*******************************************************************
*
      CONST=0.0
      SLOPE=(PU(1)-PS(N))/(DISPLA(1)-TEMP(N))
100     LOAD(J)=SLOPE*(D(J)-TEMP(N)) + ADD
       CONST= D(J)
      J=J+1
      D(J)=CONST + DELTAD
      IF (D(J) .LT. DISPLA(1)) GOTO 100
      JENDL = J-1
*
*       *       *       *       *       *
* Find the intercept point.
*       *       *       *       *       *
*
     DINT= DISPLA(1) - PU(1)/SLOPE
*
*******************************************************************
* ADD DISCRETE DATA                                               *
* TO THE                                                      *
* FAILURE REGION.                                               *
*                                                            *
*******************************************************************
      DO 110 I=2,N
       IF (D(J) .LT. DISPLA(I)) THEN
        SLOPE = PU(I)/(DISPLA(I)-DINT)
115     LOAD(J)=SLOPE*(D(J)-DINT)
        CONST = D(J)
        J=J+1
        D(J) = CONST + DELTAD
       IF (D(J) .LE. DISPLA(I)) GOTO 115
       ENDIF
110     CONTINUE
       LOAD(J)=0.0
700    PRINT *, 'CONTINOUS DATA COMPLETE '
       COUNT = J
       PRINT *, 'COUNT=',J
*
**      *       *       *       *       *
* Find the largest value for plotting
**      *       *       *       *       *
*
     CALL LARGE(K,PU,BRKLDM)
*
*       *       *       *       *       *
* Where brkldm and sldm are the maximum for that vector.
*       *       *       *       *       *
```

```
*
      DISPLM=DISPLA(N)
*
****************************************************************
*  CREATE THE NOISE
*
****************************************************************

*
      IF ((Q2 .EQ. 'Y') .OR. (Q2 .EQ. 'y')) THEN
      CALL NOISE(SEED,COUNT,LOAD,NSE)
      ENDIF
*
******************************************************************
*  PRINT THE DATA                                                *
*                                                         *
******************************************************************
      OPEN(UNIT=13,FILE='EXPER.OUT',STATUS='UNKNOWN')
      IF((Q4 .EQ. 'Y') .OR. (Q4 .EQ. 'y')) THEN
      CONST1=0.03937
      CONST2=2.2046
      ELSE
      CONST1 = 1.0
      CONST2 = 1.0
      ENDIF
*
      DO 105 I=1,COUNT
      WRITE(13,111) I,(D(I)*CONST1),(LOAD(I)*CONST2)
105     CONTINUE
      WRITE(13,116) ALPHA,BETA,N
      WRITE(13,111) COUNT,DISPLM,BRKLDM
      CLOSE(13)
******************************************************************
* FORMATS                                                      *
*                                                         *
******************************************************************
111     FORMAT(1X,I5,2X,F8.4,2X,F8.4)
116     FORMAT (1X,'PARAMETERSALPHA=',F4.1,2X,'BETA=',
     +F4.1,/,1X
     +,'N=',I4,/,1X,'N',5X,'DELTA',10X,'LOAD')
1000    FORMAT(1X,F15.1)
1500    FORMAT(20X,E16.10)
1510    FORMAT(20X,F8.4)
1515    FORMAT(20X,A6)
1520    FORMAT(20X,I5)
1530    FORMAT(1X,/)
1600    FORMAT(A1)
*
      STOP
      END
*-----------------------------------------------------------
      SUBROUTINE NOISE(SEED,COUNT,LOAD,NSE)
```

```
*-----------------------------------------------------------------
      INTEGER COUNT
      DOUBLE PRECISION SEED
      REAL LOAD(COUNT),NSE,RNDM
      DO 10 I=1,COUNT
      CALL RANDN(SEED,RNDM)
      LOAD(I) = LOAD(I) + (RNDM*NSE - NSE/2)
10     CONTINUE
      END
*-----------------------------------------------------------------
      SUBROUTINE SWITCH(N,ORDER,ARRAY)
*-----------------------------------------------------------------
      INTEGER N,ORDER(N),LIM1
      PARAMETER(LIM=5000)
      REAL ARRAY(N),TEMP(LIM)
      DO 1 I=1,N
       TEMP(I)=ARRAY(I)
1      CONTINUE
*
      DO 2 I=1,N
       K=ORDER(I)
       ARRAY(I)=TEMP(K)
2     CONTINUE
      END
*
*-----------------------------------------------------------------
      SUBROUTINE SWCH2(N,ORDER,ARRAY)
*-----------------------------------------------------------------
      INTEGER N,LIM1
      PARAMETER(LIM1=5000)
      INTEGER ARRAY(N),TEMP(LIM1),ORDER(N)
      DO 1 I=1,N
       TEMP(I)=ARRAY(I)
1      CONTINUE
*
      DO 2 I=1,N
       ARRAY(I)=TEMP(ORDER(I))
2     CONTINUE
      END
*
```

```
*---------------------------------------------------------------
      SUBROUTINE  LARGE(MAX,VECTOR,MAXVAL)
*---------------------------------------------------------------
      INTEGER MAX
      REAL VECTOR(MAX),MAXVAL
      MAXVAL=VECTOR(1)
      DO 10 J=2,MAX
      IF (VECTOR(J) .GT. MAXVAL) THEN
      MAXVAL = VECTOR(J)
      ENDIF
10     CONTINUE
      END
*  ---------------------------------------------------------------
      SUBROUTINE  SMALL(TOTAL,VECTOR,LOW)
*---------------------------------------------------------------
*  THIS ROUTINE TAKES A MATRIX IN VECTOR FORM AND
*  FINDS THE SMALLEST
*  VALUE IN EACH ROW AND PUTS IT INTO A VECTOR
     INTEGER TOTAL,ROWS,COLS
     REAL VECTOR(TOTAL),LOW
     COMMON /SML/ ROWS,COLS
* LOOK AT ONLY ROWS
       LOW=VECTOR(1)
*
      DO 10 I=1,ROWS
       IF (VECTOR(I) .LT. LOW) LOW=VECTOR(I)
*
10    CONTINUE
     END
*---------------------------------------------------------------
     SUBROUTINE  SHELL(MAXNUM,ARRAY,NO)
*---------------------------------------------------------------
*     MAXNUM IS THE NUMBER IN THE VECTOR TO BE
*     SORTED
     INTEGER MAXNUM,OFFSET,SW,NO,LIMIT
     DIMENSION NO(MAXNUM)
     REAL ARRAY(MAXNUM),TEMP,TEMP2
     LOGICAL SWITCH
*
      PRINT *, 'SUB SHELL'  OFFSET=MAXNUM/2
1     IF (OFFSET .GT. 0) THEN
LIMIT=MAXNUM-OFFSET
5       SWITCH= .FALSE.
*
          DO 15 J=1,LIMIT
            IF (ARRAY(J) .GE. ARRAY(J+OFFSET)) THEN
              TEMP = ARRAY(J)
              TEMP2 = NO(J)
             ARRAY(J)=ARRAY(J+OFFSET)
              NO(J)=NO(J+OFFSET)
             ARRAY(J+OFFSET)=TEMP
              NO(J+OFFSET)=TEMP2
```

```
          SWITCH = .TRUE.
          SW=J
         ENDIF
15      CONTINUE
       LIMIT = SW -OFFSET
       IF(SWITCH) GOTO 5
        OFFSET=OFFSET/2
        GOTO 1
       ENDIF
*
      END
*
* ------------------------------------------------------------
     SUBROUTINE RAND(ASEED,TOTAL,RNDM)
* ------------------------------------------------------------
*    Random number generator - Uniformly distributed in (0,1)
*       SEED in (1,247483647)
     DOUBLE PRECISION RANDM,ASEED
     INTEGER TOTAL
     REAL RNDM(TOTAL)
*
     DO 20 I=1,TOTAL
     ASEED=ASEED+1.0
     RANDM = DMOD(16807.0D0*ASEED,2147483647.0D0)
     ASEED=RANDM
     RNDM(I) = SNGL(RANDM*4.6566128752458D-10)-1.0E-07
20    CONTINUE
     ASEED=ANINT(ASEED)
END
* ------------------------------------------------------------
     SUBROUTINE RANDN(ASEED,RNDM)
* ------------------------------------------------------------
*    Random number generator - Uniformly distributed in (0,1)
*       SEED in (1,247483647)
     DOUBLE PRECISION RANDM,ASEED
     INTEGER TOTAL
     REAL RNDM
     ASEED=ASEED+1.0
     RANDM = DMOD(16807.0D0*ASEED,2147483647.0D0)
     ASEED=RANDM
     RNDM = SNGL(RANDM*4.6566128752458D-10)-1.0E-07
     ASEED=ANINT(ASEED)
      END
*-----------------------------------------------------------
     SUBROUTINE NORMAL(ALPHA,BETA,N,RAND2,
    +SLACK,MAXSLK)
*  ----------------------------------------------------------
*    PERFORMS A SIMULATION OF EXPERIMENTS USING THE *
*    WEIBULL MODEL
*
     INTEGER N
     REAL ALPHA,BETA,MEAN,MAXSLK,RAND2(N),
```

```
     +SLACK(N),RNDM
        MAXSLK = 0.0
        PRINT *, 'SUB NORMAL'
*-- CALCULATE THE STRENGTH OF THE WEIBULL FUNCTION,
       DO 10 I=1,N
        RNDM = RAND2(I)
                     S L A C K ( I )    =
+EXP((LOG(-LOG(1-RNDM))+ALPHA*LOG(BETA))/ALPHA)
        IF (SLACK(I) .GT. MAXSLK) THEN
        MAXSLK = SLACK(I)
        ENDIF
 10     CONTINUE
      END
*     END
*-----------------------------------------------------------
      SUBROUTINE
     + UNIFM(RANGE1,RANGE2,N,RAND2,SLACK,MAXSLK)
*-----------------------------------------------------------
      INTEGER N
      REAL RANGE,MEAN,RAND2(N),SLACK(N),MAXSLK
*
      CONST=RANGE2-RANGE1
      MAXSLK=0.0
      DO 10 I=1,N
      SLACK(I)=RAND2(I)*CONST + RANGE1
      IF (MAXSLK .LT. SLACK(I)) THEN
      MAXSLK=SLACK(I)
      ENDIF
10     CONTINUE
      END
*  ---------------------------------------------------------
      SUBROUTINE WEIBUL(ALPHA,BETA,TOTAL,BUNDLE,P)
*  ---------------------------------------------------------
*     PERFORMS A SIMULATION OF EXPERIMENTS USING THE
*     WEIBULL MODEL
*     INITIALIZE
      REAL ALPHA,BETA
      INTEGER TOTAL,MAX
      REAL P(TOTAL),BUNDLE(TOTAL),RNDM
    PRINT *, 'SUB WEIBUL'
*-- CALCULATE THE STRENGTH OF THE WEIBULL FUNCTION,
      DO 10 I=1, TOTAL
         RNDM = BUNDLE(I)
      P(I) =
     + EXP((LOG(-LOG(1-RNDM))+ALPHA*LOG(BETA))/ALPHA)
 10     CONTINUE
      END
```

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